Mathematics 116:
Convexity and Optimization with Applications

Themes

This course is really about the **triumphs and failures** of calculus and linear algebra in infinite dimensions. So the first thing we will have to look into is **why anyone would care** about infinite dimensional spaces. Not just mathematicians, either, but economists, physicists, engineers, chemists, biologists, statisticians, and others who **think big** and have broad horizons. Then we will have to look at **what's so good about calculus** and linear algebra in two or three dimensions that seems worth trying to generalize. If all goes well, we will end up not only developing key ideas of **real and functional analysis**, but also addressing some truly significant applications ranging from growth, price, and game theories in economics to mechanics and field theories in physics, and from financial planning to soap bubbles. The course is designed to facilitate participation by students with a **wide variety of backgrounds and interests**. See http://www.courses.fas.harvard.edu/~math116/

Audience

Mathematics 116 welcomes **both undergraduates and graduate students**. The grading scheme insures that everyone who works hard and eventually learns the basics can earn an A. Pursuing a **pure or applied** project is optional but highly encouraged. The official prerequisite is at least one course beyond Mathematics 21, but potential students are also invited to contact the instructor about their experience, interests, and **goals**.

Instructor

Daniel Goroff, Professor of the Practice of Mathematics.
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Science Center 427, especially Tuesdays 1:00 - 3:00.

Meetings

Lectures **Tuesdays and Thursdays 11:30-1:00** in Science Center 216.
Problem sessions will meet weekly at a time to be arranged.

Texts

A First Course in Optimization Theory by Ranagarajan K. Sundaram includes lots of good examples and motivation.
Optimization by Vector Space Methods is a classic by David Luenberger. There are many more references we can discuss, too, depending on particular interests and purposes.

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Topics

I. **Goals and Examples** (see FCOC 2; OVSM 1)
Applications in finite and infinite dimensions.

II. **Existence of Optima** (FCOC 3; OVSM 2, 5, 6)
Continuity, Compactness; Banach Spaces, Duals.

III. **Necessary Conditions** (FCOC 4, 5; OVSM 7, 9)
Derivatives and Lagrange Multipliers.

IV. **Sufficient Conditions** (FCOC 7; OVSM 7)
Convexity, Concavity; Separating Hyperplanes.

V. **Parametrized Problems** (FCOC 6, 9; OVSM 8)
Kuhn-Tucker and Envelop Theorems.

VI. **Duality** (OVSM 8)
Fenchel and Rockafellar Theorems.

VII. **More Applications** (FCOC 11,12; OVSM 9)
Dynamic Programming; Control and Maximum Principle.

Dates

Midterm planned for April 7. Optional projects are due May 12.
Optional presentations but no new lectures during Reading Period.

Grades

Out of a fixed **target** of 500 points, it is expected but not promised that students who end up with more than 450 will earn an **A** of some kind, that students who end up with between 400 and 450 will earn a B of some kind, etc. The final examination will count for the **difference** between the 500 point target and the number of points you have accumulated during the term. This allows unlimited **resurrection** for those who might take longer to master the material.

Points

You can earn up to a possible total of 420 points during the term. There will be eight **problems sets** worth 30 points each. You should consider these mandatory, but in principle everything but the final is **optional**. Other ways of earning points include: (1) taking the **midterm**, which is worth 60 points; (2) completing a theoretical or applied **project** about ten pages long or less reporting on a topic related to the course and worth up to 60 points; or (3) **posting** to the course site up to four brief presentations connecting course topics with your interests worth 10 points each, and/or posting up to four reactions worth five points each.

Final

The three hour final examination will be given during the second half of May on a date determined by the Registrar. It will have three parts: (1) questions very similar to or **the same as homework** problems; (2) **straightforward questions** that combine or test key ideas from the course; and (3) an **essay** summarizing your project or on another topic that you propose and prepare in advance.