Mathematics 116
Convexity and Optimization with Applications

Project Guidelines and Suggestions

Projects are a chance for you to consolidate and apply what we have been learning together. Throughout the semester, we have developed many powerful concepts and results. With these tools, you are now well equipped to explore applications or extensions of these ideas that are of particular interest to you.

As it says in the syllabus, projects are meant to be five to ten pages or so. Roughly speaking, a project should take you a little more time to complete than a typical problem set (since it counts for twice as many points) but not much more.

Please do pick a topic that both interests you and that you can relate directly to at least one theorem in Luenberger or Sundaram. As always, I am glad to discuss ideas with you and suggest other resources. See also the options that follow.

Write up your project as an essay report to the extent possible. In particular, start with a paragraph or more that introduces your subject, that says something about why it is interesting to you, and that outlines what you are going to do.

Ideally, you should find and read more than one source that discusses your topic. As with any academic work, please cite your references, be they books, web pages, people, or whatever. Make sure you explicitly relate your topic to at least one theorem in our textbooks. Write about what that theorem says, how you have come to believe it, and why it is significant in your own words. You may want to compare and contrast the approach, notation, level of generality, etc. of various authors.

Filling in the details of arguments you have read would be very appropriate to include. So would examples, exercises, illustrations, calculations, or computer programs that help illuminate your topic, preferably ones you make up yourself but also ones suggested to you by books or elsewhere.

Projects are due on Thursday, May 12, in the mailbox labeled Math 116 outside Science Center 325. Graded projects and homework will be in that box for you to pick up before the final examination, which is presently scheduled by the Registrar for the afternoon of Monday, May 23.

If, before May 10, you post your project to the discussion section of our website (or even just a good summary of it) we will all be able to talk about it on line or in class, and you will earn extra participation points, as usual.

Here are some possible topics that may not involve looking very far:

1. Expand on what Luenberger says in Chapter 3 about the convergence of Fourier series or other approximation schemes, and investigate more applications or exercises from §3.13.

2. Go through Chapter 4 of Luenberger, describe more instances where estimation or least squares come up in statistics, work more exercises from §4.8. Of particular interest and importance in engineering is the idea of the Kalman Filter introduced in §4.7.

3. Study Chapter 8 of Luenberger and Chapter 7 of Sundaram, find more instances of how Lagrange Multipliers appear in economics, in the Kuhn-Tucker Theorem, or elsewhere, then work more exercises from §8.8 of OVSM and §7.8 of FCOT.

4. Study Chapter 9 of OVSM, find more treatments and applications of control theory in engineering that interest you, work a few exercises from§9.7.

5. Study Chapter 10 of OVSM, find more descriptions and uses in computer science of at least one algorithm there, work a few exercises from §10.12.

6. Discuss the Principle of Least Action in classical mechanics, particularly how Newton's Laws follow from the Euler-Lagrange equations, and also how the Legendre transform relates to the duality we have developed.

7. Discuss the duality of linear programming problems, and work out how this is a special case of the §7.12. The book on *Convex Analysis and Nonlinear Optimization* by Borwein and Lewis has more about this and some exercises, too, for example.

8. Read what Borwein and Lewis or others have to say about how the subdifferential of a convex function is a kind of generalized derivative that works even when the function is not differentiable. Compare this with Luenberger's exercise 8 in §8.8. Discuss applications. Work some of Borwein's exercises.

9. Describe what economists call the Envelop Theorem and relate it to §8.5 of Luenberger. Discuss heuristic proofs and why they fail, etc. Compare with Sundaram's Maximum Theorem in his Chapter 9.

10. Expand on how what Riesz Representation Theorem in §5.5 of OVSM can tell you about measure theory in general and about probability measures in particular.

11. Solve Steiner's Problem, look up generalizations to networking problems or other related problems, calculate and discuss the dual of Steiner's problem.

12. Investigate further the Ramsey model of economic growth and planning.

13. Study Chapters 11 and 12 of Sundaram on dynamic programming, work some more problems, and try relating the discrete time approach there to Control Theory with continuous time as in §9.5 of Luenberger.