

Mathematics 116

Convexity and Optimization with Applications

Assignment VII	Due in class on April 28
Announcements	Please go over your Midterm Examinations and, as always, ask questions about anything that still needs clarifying.
Reading	Chapter 6 and Chapter 7 of OVSM.
Exercises	Do three of these from OVSM §6.12: #5, 6, 12, 14, 15. Do three of these from OVSM §7.14: #1,4, 6, 10.

A. With the conditions $x(0)=1$ and $x(1)=2$, find extremals for the functional:

$$\int_0^1 \{ [x'(t)]^2 + 10tx(t) \} dt .$$

B. Suppose a mine has an amount B of some resource. The profit rate that can be earned by selling that resource at rate x is $\ln(x)$. Find the rate at which the resource should be sold over a fixed period $[0, T]$ to maximize the present value of profits from the mine. Assume the discount rate is a constant r and that the resource has no value beyond time T . [Hint: If you let $y(t)$ denote the cumulative amount sold by time t , then $y'(t)$ is the sales rate, so find $y(t)$ to maximize the integral of $e^{-rt} \ln y'(t)$ subject to $y(0)=0$ and $y(T)=B$.]

Writing

Let $w_t(a) = 0$ and, for t in $[a,b]$, suppose $w_t(s)$ is zero for $s < t$ and equals 1 for $s > t$. Considered as an element of $\text{NBV}[a,b]$, describe how w_t determines an element of the dual space of $C[a, b]$ by showing what it does to a typical continuous function x on $[a,b]$. What is the norm of w_t ? Now suppose that t_n is a sequence in $[a, b]$ converging to t_0 . What can you say about the sequence w_{t_n} ? Does it converge strongly? Is it weak-* convergent? How does your answer relate to Alaoglu's Theorem? Do you think linear combinations of functions of the form w_t are dense in $\text{NBV}[a,b]$? Weak-* dense? Discuss and illustrate your guesses, but you need not provide proofs.

Discussion

Talk about and post your ideas for a project based on the handout.