Mathematics 116:
Convexity and Optimization with Applications

"Nothing at all takes place in the universe in which some rule of maximum or minimum does not appear." -L. Euler (1707-1783)

Themes

This course is really about the **triumphs and failures** of calculus and linear algebra in infinite dimensions. So the first thing we will have to look into is **why anyone would care** about infinite dimensional spaces. Not just mathematicians, either, but economists, physicists, engineers, chemists, biologists, statisticians, and others who **think big** and have broad horizons. Then we will have to look at **what's so good about calculus** and linear algebra in two or three dimensions that seems worth trying to generalize. If all goes well, we will end up not only developing key ideas of **real and functional analysis**, but also addressing some truly significant applications ranging from growth, price, and game theories in economics to mechanics and field theories in physics, and from financial planning to soap bubbles. The course is designed to facilitate participation by students with a **wide variety of backgrounds and interests**. See http://www.courses.fas.harvard.edu/~math116/

Audience

Mathematics 116 welcomes **both undergraduates and graduate students**. The grading scheme insures that everyone who works hard and eventually learns the basics can earn an A. Pursuing a **pure or applied** project is optional but highly encouraged. The official prerequisite is at least one course beyond Mathematics 21, but potential students are also invited to contact the instructor about their experience, interests, and **goals**.

Instructor

Daniel Goroff, Professor of the Practice of Mathematics.
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Science Center 427, especially Weds 3:30-5 and Thurs 1-2:30.

Meetings

Lectures **Mon and Weds 2-3:30** in Science Center 216.
Save Fridays at 2 for problem sessions or other activities there.

Texts

Optimization by Vector Space Methods is a required classic by David Luenberger. Also recommended for the more technically inclined is **Convex Analysis and Nonlinear Optimization** by Jonathan Borwein and Adrian Lewis. There are lots more references we can discuss, too, depending on particular interests and purposes.

Topics	I. Goals and Examples	(see text chapter 1)
	II. Who Cares about Infinite Dimensions	(see text chapter 2)
	III. Geometry of Infinite Dimensions	(see text chapter 3)
	IV. Approximations and Least Squares	(see text chapter 4)
	V. Planes, Hyperplanes, and Duality	(see text chapter 5)
	VI. But I Used to Like Matrices	(see text chapter 6)
	VII. Derivatives Again	(see text chapter 7)
	VIII. Constraints and Sensitivity	(see text chapters 8, 9)
	IX. More Applications	(supplemental material)

Dates

No lecture on October 6, October 13, or November 26.
Potential make-up meetings to be announced.
Midterm planned for the week of November 10.
Optional presentations but no new lectures during Reading Period.
Optional projects are due January 9.

Grades

Out of a fixed **target** of 500 points, it is expected but not promised that students who end up with more than 450 will earn an **A** of some kind, that students who end up with between 400 and 450 will earn a B of some kind, etc. The final examination will count for the **difference** between the 500 point target and the number of points you have accumulated during the term. This allows unlimited **resurrection** for those who might take longer to master the material.

Points

You can earn up to a possible total of 420 points during the term. There will be eight **problems sets** worth 30 points each. You should consider these mandatory, but in principle everything but the final is **optional**. Other ways of earning points include: (1) taking the **midterm**, which is worth 60 points; (2) completing a theoretical or applied **project** about ten pages long or less reporting on a topic related to the course and worth up to 60 points; or (3) **posting** to the course site up to four brief presentations connecting course topics with your interests worth 10 points each, and/or posting up to four reactions worth five points each. You are encouraged to discuss homework with others, but you must write up solutions yourself.

Final

The three hour final examination will be given during the second half of January on a date determined by the Registrar. It will have three parts: (1) questions very similar to or **the same as homework** problems; (2) **straightforward questions** that combine or test key ideas from the course; and (3) an **essay** summarizing your project or on another topic that you propose and prepare in advance.