Mathematics 116
Convexity and Optimization with Applications

Assignment VI Due in class on November 17.

Announcements In addition to office hours and sections, help is also available on Sundays at 8pm at the Math Question Center in Loker Commons. Sections are Thursday at 5:00 in SC116 and Friday at 2:00 in SC216. Note: The midterm will be in class on Wednesday, December 3.

Reading Chapters 4 and 5

Exercises Do the following from Luenberger §5.14: #1, 2.

Writing Include answers to five of these with your problem set. [Dorny]

1. This problem extends what we discussed in class slightly to multivariable linear regression. Suppose a planner has developed the following workforce model for a government agency:

$$f(\square_1, \square_2) = b_0 + b_1\square_1 + b_2\square_2$$

where f is the total workforce in the agency, b_0 is the person power required for overhead (payroll, building maintenance, planning, etc.), $\square_1$ is the number of grant applications processed in a year, b_1 is the number of person-years required to process one application, $\square_2$ is the number of technical consultations provided by the agency in a year, and b_2 is the person power required to provide one consultation. The quantities b_0, b_1, and b_2 are measures of the productivity of the agency. The following historical data is available from the agency:

Year	Employees	Applications Processed	Consultations Provided
1	93	5804	1750
2	100	6250	1878
3	102	6200	1852
4	105	6225	1860

(a) Describe how to determine those values of b_0, b_1, and b_2 for which the model best fits this data in a least-square sense.
(b) Compute the solution. A calculator or computer may help.

2. Let **A** be a real *n* by *n* matrix. Assume the standard inner product on the space of all such matrices, (**A** | **B**) = *trace* ($\mathbf{B}^T$ **A**), where *trace* (**C**) is the sum of the diagonal entries of **C**. Find the "constant matrix" of the form *c***I** which best approximates **A** in the least square sense.

3. Find the least square error solution for the following system:

$$\alpha_1 + 2\alpha_2 = 3$$
$$2\alpha_1 + \alpha_2 = 3$$
$$4\alpha_1 + 5\alpha_2 = 10$$

and describe what your answer means geometrically.

4. This problem also extends slightly an example discussed in class. Suppose a system puts out a sequence of electrical pulses of magnitude x_i where $0 \le x_i \le 2$. The probability density for these pulses is unknown. However, observation of past pulses has shown that the average values of x_i , x_i^2, $x_i x_{i-1}$, and $x_i x_{i-2}$ are 1, 9/4, 7/4, and 5/4 respectively. Estimate the future pulse height x_n by an affine function of the two previous pulse heights:

$$\hat{x}_n = \alpha_0 \mathbf{1} + \alpha_1 x_{n-1} + \alpha_2 x_{n-2}$$

Pick multipliers α_0,α_1, and α_2 to minimize the expected mean-square error, $E[(x_n - \hat{x}_n)^2]$. (Note that α_0 multiplies a random variable whose sample values are always 1.)

4. Find the continuous function *x* for which $\int_0^1 x(t)dt = 1$ and $\int_0^1 x^2(t)dt$ is minimum.

5. Define a new inner product on $L_2[0, 1]$ by setting

$$(x \mid y) = \int_0^1 e^t x(t) y(t) dt$$

Using this "weighted" inner product to define a norm, reconsider Example 1 on page 66 of Luenberger and find the control voltage function *u* that solves the motor moving problem given there but minimizes this norm. Explain what effect this change has on the previous energy minimizing solution.

Discussion

For section. You are also encouraged to post what you find to the discussion section of our website, along with any questions or observations you might also have.

1. Show that C[0,1] with the usual norm is not an inner product space. (Hint: Find simple functions that violate the parallelogram equality.)

2. Theorem 1 of §4.7 describes Kalman's Recursive Estimation Procedure. This is an important technique in many applications. Without going through all the equations, can you explain in your own words what is going on, how it relates to the ideas about projections we have developed, and why anyone would care? You might try looking up the "Kalman Filter" on the web for ideas.