Mathematics 116
Convexity and Optimization with Applications

Assignment IV — Due in class on Monday, October 27.

Announcements — In addition to office hours and sections, help is also available on Sundays at 8pm at the Math Question Center in Loker Commons.

Reading — Chapters 3 and 4 of Luenberger.

Exercises — Do five from among Luenberger §3.13 #1, 3, 4, 5, 6, 9, 24.

Writing — Please include your answers with your problem set.

1. Let Y be the set of sequences $x = (\xi_1, \xi_2, ...)$ with all but finitely many coordinates equal to zero. Show that this is a subspace of ℓ that is not closed when using the usual norm on ℓ. What is its closure with respect to that norm? What does this tell you about the completion of Y with respect to that norm? What can you say about completing Y with respect to the p-norm $\| x \|_p = (\sum_{i=1} |\xi_i|^p)^{1/p}$? (c.f. Example 4 on pp. 36-37.)

2. Give examples that illustrate how ℓ_1 is a proper subset of ℓ_2. Explain in your own words why you should expect this to be true and how this fact generalizes. What about L_2 [0,1] and L_1[0,1]?

Discussion — For discussion in section. You are also encouraged to post what you find to the discussion section of our website, along with any questions or observations you might also have.

1. Find and describe in a paragraph or so a theoretical or applied situation which makes use of one of the following ideas: Least Squares Approximation; Gram-Schmidt Procedure; Fourier Seriers; or Legendre Polynomials, Laguerre Polynomials, or other special functions that can be regarded as providing an orthonormal basis for an appropriate space.

2. What favorite ways from calculus do you have for showing that the harmonic series 1 + 1/2 + 1/3 + … diverges but the series 1 + 1/4 + 1/9 + 1/16 + …. converges? A surprising result due to Euler is that the latter actually adds up to $\pi^2/6$. Can you imagine or look up a convincing justification for this?