

Name: _____

In Class MidTerm Exam for Math 113

10 - 11:30 am, March 20, 2008

Problem	Points	Score
1	15	
2	15	
3	14	
4	15	
5	12	
6	14	
7	15	
Total	100	

- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- Some problems will be asking you to prove results covered in class. For those that do not, you may quote without proof the theorems/propositions/lemmas given during lecture as long as you state each result clearly. Points may be deducted for incorrect or missing hypotheses.
- **NO** calculators are permitted.

1. (15 points) Simplify the following expressions:

(a) $e^{\log i}$

(b) $\log i$

(c) $i^{\log(-1)}$

2. (15 points)

- (a) Prove that the n nth roots of unity can be expressed as $1, w, w^2, w^3, \dots, w^{n-1}$.
- (b) What is w ?
- (c) Show that the sum of the n nth roots of unity is zero.

3. (a) (7 points) Is $f(z) = \bar{z}$ analytic? Prove or disprove.
- (b) (7 points) Let A be an open subset of \mathcal{C} and $A^* = \{z : \bar{z} \in A\}$. Suppose f is analytic on A and define a function g on A^* by

$$g(z) = \overline{f(\bar{z})}.$$

Show that g is analytic on A^* .

4. (15 points)

- (a) Define what it means for a function to be analytic (holomorphic) at a point $z \in \mathbb{C}$.
- (b) State the Cauchy-Riemann equations.
- (c) Prove that a function analytic at a point $z \in \mathbb{C}$ satisfies the Cauchy-Riemann equations at that point.

5. Evaluate the following integrals with γ being the unit circle centered at the origin:

(a) (3 points) $\int_{\gamma} \sin z \, dz.$

(b) (3 points) $\int_{\gamma} \frac{\sin z}{z} \, dz.$

(c) (3 points) $\int_{\gamma} \frac{\sin z}{z^2} \, dz.$

(d) (3 points) $\int_{\gamma} \frac{\sin(e^z)}{z^2} \, dz.$

6. (14 points) Evaluate $\int_0^{2\pi} e^{-i\theta} e^{e^{i\theta}} d\theta$.

7. (15 points) Let f be entire and let $|f(z)| \leq M$ for z on the circle $|z| = R$ with R fixed. Prove that

$$\left| f^{(k)}(z) \right| \leq \frac{k!M}{(R-r)^k}, \quad r = |z|,$$

for all $0 \leq r < R$ and for all $k = 0, 1, 2, 3, \dots$

Extra space for work: