Problem 1 (§1.1, 16). For any k,

$$i^{4k} = (i^4)^k = ((i^2)^2)^k = ((-1)^2)^k = 1^k = 1.$$

From this, we can also conclude that

$$i^{4k+1} = (i^{4k}) \cdot i = 1 \cdot i = i,$$

$$i^{4k+2} = (i^{4k+1}) \cdot i = i \cdot i = -1,$$

and

$$i^{4k+3} = (i^{4k+2}) \cdot i = -1 \cdot i = -i.$$

We conclude that $i^n = 1$ if $n \equiv 0 \mod 4$, $i^n = i$ if $n \equiv 1 \mod 4$, $i^n = -1$ if $n \equiv 2 \mod 4$, and $i^n = -i$ if $n \equiv 3 \mod 4$. This specifies i^n for all $n \in \mathbb{Z}$.

Problem 2 (§1.1, 17). a) We compute that

$$(1+i)^4 = ((1+i)^2)^2 = (1^2 + 2 \cdot 1 \cdot i + i^2)^2 = (1 + 2i - 1)^2 = (2i)^2 = 4 \cdot i^2 = -4.$$

b) Similarly,

$$(-i)^{-1} = \frac{1}{-i} = \frac{1}{-i} \cdot \frac{i}{i} = \frac{i}{-i^2} = \frac{i}{1} = i.$$

Problem 3 (§1.2, 18). a) Assume that $z \neq 0$, so that $\arg z$ is defined. Recall that $z\bar{z} = |z|^2 \in \mathbb{R}_{\geq 0}$; thus $0 = \arg(z\bar{z}) = \arg z + \arg \bar{z} \pmod{2\pi}$. So $\arg \bar{z} = -\arg z \pmod{2\pi}$, as desired.

b) Suppose that $w, z \neq 0$. Using the previous part of this exercise and the observation that $1/w = \bar{w}/|w|^2$ (since $w\bar{w} = |w|^2$), we have

$$\arg(z/w) = \arg(z\bar{w}/|w|^2) = \arg z + \arg \bar{w} + \arg(1/|w|^2) = \arg z - \arg w + 0 = \arg z - \arg w \pmod{2\pi}.$$

c) We claim that $|z| = 0$ if and only if $z = 0$. The "if" direction is immediate, since $|0| = 0$. On the other hand, suppose that $|z| = 0$ where $z = a + bi$, with $a, b \in \mathbb{R}$. Then $|z| = \sqrt{a^2 + b^2} = 0$, so $a^2 + b^2 = 0$. Since for real numbers x we have $x^2 \geq 0$, with equality exactly when $x = 0$, we conclude that $a = b = 0$, and thus $z = 0$, completing the proof.