MATH 113 – PRACTICE PROBLEMS FOR THE MIDTERM
MARCH 11, 2005

(1) (a) Is $f(z) = z^2$ holomorphic on $\mathbb{C}$? Why or why not?
(b) Is $f(z) = |z^2|$ holomorphic on $\mathbb{C}$? Why or why not?

(2) Let $f(z) = z^2$ and $g(z) = e^z$. What is the image of the domain

$$\{z \ : \ |z| > 0 \text{ and } 0 < \ \arg z \ < \pi/4\}$$

under the composed map g(f(z))? Draw the picture and describe the set.

(3) Find the domains on which the function

$$f(z) = |x^2 - y^2| + 2i|xy|$$

is analytic.

(4) Does the function $u(z) = \log |z - 1|$ have a harmonic conjugate on either of the following domains? If so, provide one, and if not please explain why.
(a) the annulus $\{z \ : \ 1 < |z| < 2\}$.
(b) the disk $\{z \ : \ |z| < 1\}$.

(5) Evaluate the following integrals.
(a) $\int_{|z|=1} \overline{z} dz$.
(b) $\int_{|z|=\rho} \frac{z^5}{z-2} dz$, both when $0 < \rho < 2$ and when $2 < \rho < \infty$.
(c) $\int_\gamma \frac{e^z}{z(1-z)^3} dz$, where γ is a simple closed curve such that
  (i) the point 0 lies inside and the point 1 lies outside of the domain bounded by γ.
  (ii) the point 1 lies inside and the point 0 lies outside of the domain bounded by γ.
  (iii) both 0 and 1 lie inside the domain bounded by γ.

(6) Suppose $f : \mathbb{C} \to \mathbb{C}$ is entire (i.e. analytic on $\mathbb{C}$) and satisfies

$$f(z) = f(z + 1) = f(z + i)$$

for all $z \in \mathbb{C}$, and $f(0) = 1$. What is f and why?

(7) (a) Expand the function $f(z) = \frac{z}{z+2}$ as a power series around the point $z = 1$, and find the radius of convergence.
(b) Find the first five terms of the power series expantion of $f(z) = (1+z)^z$ around $z = 0$.

(8) Prove that the distance between the point $z = 0$ and the zero of the function $f(z) = \sum a_k z^k$ nearest to it is not less that $\frac{\rho|a_0|}{M+|a_0|}$, where ρ is any number not exceeding the radius of convergence of the series, and $M = M(\rho) = \max_{|z|=\rho} |f(z)|$.