Mathematics 113 Fall 2001

Homework Assignment # 2

Due Thursday, September 27

1. Find all complex solutions of $e^z = -5$.

2. Find all complex solutions of $\sin(z) = 5$.

3. If g is an analytic function on the open set $A \subseteq \mathbf{C}$ and $B := \{z \in A \mid g(z) \neq 0\}$, show that B is open and that $1/g$ is analytic on B.

4. Define the symbols $\partial f/\partial z$ and $\partial f/\partial \overline{z}$ by

$$\partial f/\partial z := \frac{1}{2}(\partial f/\partial x + \frac{1}{i}\partial f/\partial y).$$

and

$$\partial f/\partial \overline{z} := \frac{1}{2}(\partial f/\partial x - \frac{1}{i}\partial f/\partial y).$$

Show that the Cauchy–Riemann equations are equivalent to $\partial f/\partial \overline{z} = 0$, and that if f is analytic at z_0, then $f'(z_0) = \partial f/\partial z(z_0)$.

5. Show that if $f, g : \mathbf{C} \to \mathbf{C}$ are entire, then $g \circ f$ is entire and

$$(g \circ f)'(z) = g'(f(z)) \cdot f'(z).$$

[You may want to look at the hint on page 41 of the book.]