

Math S-101. Worksheet 10.

Connected Sets

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Connected Sets

Two sets A and B in X are *separated* if

$$\mathbf{K}(A) \cap B = A \cap \mathbf{K}(B) = \emptyset.$$

Any subset $W \subset X$ is *disconnected* if there are nonempty, separated sets A and B such that $W = A \cup B$. If no such sets exist, then W is *connected*.

Theorem 1 *Two sets A and B separate X if and only if $A \cup B = X$ and $A \cap B = \emptyset$, where A and B are nonempty closed subsets of X .*

Problems

1. If $a < b < c$, show that the set $(a, b) \cup (b, c)$ is disconnected in \mathbb{R} .
2. Two sets A and B separate X if and only if $A \cup B = X$ and $A \cap B = \emptyset$, where A and B are nonempty closed subsets of X .
3. Suppose that A and B separate X and $Y \subset X$. If $A \cap Y \neq \emptyset$ and $B \cap Y \neq \emptyset$, show that $A \cap Y$ and $B \cap Y$ separate Y .