

Math S–101. Worksheet 4.

Logic

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Summer 2006

Some Definitions

- A *proposition* is any declarative statement that is true or false.
- The following symbols are called *connectives* and are used to stand for words.
 - \wedge for “and”
 - \vee for “or”
 - \sim for “not”
 - \rightarrow for “implies” or “if ... then”
 - \leftrightarrow for “if and only if”
- *Truth functions* for the connectives are defined as follows.
 - $P \wedge Q$ is true provided both P and Q are true.
 - $P \vee Q$ is true provided at least one of the statements P and Q are true.
 - $\sim P$ is true provided P is false.
 - $P \rightarrow Q$ is true provided P is false or Q is true (or both).
 - $P \leftrightarrow Q$ is true provided P and Q are both true or both false.

We can represent these truth functions in *truth tables*.

- A *tautology* is the has all Ts as outputs. A *contradiction* is a statement that has all Fs as outputs. $(\sim P \rightarrow Q) \leftrightarrow (P \vee Q)$ is an example of a tautology. Two statements are *propositionally equivalent* if a tautology results when the connective \leftrightarrow is put between them.

- Given any conditional $P \rightarrow Q$,
 - the statement $Q \rightarrow P$ is called its *converse*.
 - the statement $\sim P \rightarrow \sim Q$ is called its *inverse*.
 - the statement $\sim Q \rightarrow \sim P$ is called its *contrapositive*.

Problems

1. Which of the following are propositions?
 - (a) 10 is a prime number.
 - (b) Are there any even prime numbers?
 - (c) Turn off that music or I'll scream.
 - (d) Life is good.
 - (e) $3 + 5$
 - (f) The number π is bigger than 4.
 - (g) Benjamin Franklin had many friends.
 - (h) The Chicago Cubs will win the World Series in 2016.
 - (i) I like olives but not very much.
 - (j) Goldbach's conjecture is true.

2. Construct truth tables for each of the following statements

(a) $\sim (P \wedge Q)$	(d) $P \leftrightarrow \sim P$
(b) $P \leftrightarrow (P \vee Q)$	(e) $P \rightarrow (Q \rightarrow (P \wedge Q))$
(c) $P \rightarrow \sim P$	(f) $\sim (P \wedge Q) \rightarrow (\sim P \wedge \sim Q)$

3. Determine which of the following pairs of statements are propositionally equivalent.

(a) P and $\sim \sim P$	(d) $P \leftrightarrow Q$ and $Q \rightarrow P$
(b) $\sim (P \vee Q)$ and $\sim P \vee \sim Q$	
(c) $\sim (P \vee Q)$ and $\sim P \wedge \sim Q$	(e) $P \leftrightarrow Q$ and $\sim Q \rightarrow \sim P$