Name:______________________________

Math S–101. Midterm 1—Tuesday, July 18, 2006

Problem Number	Possible Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
Total	50	

Please Read. You have 90 minutes to take this exam. Do any five of the following problems. Please be sure to write neatly in the exam booklet—illegible answers will receive little or no credit. If you do more than five problems, please indicate which problems that you would like us to read by crossing out the remaining solutions. If you fail to do so, only the first five problems in your exam booklet will be read. Any additional problems will be ignored even if they are correct.

Be sure to use correct mathematical notation. Any answer in decimal form must be accurate to three decimal places, unless otherwise specified. To receive full credit on a problem, you will need to justify your answers carefully—unsubstantiated answers will receive little or no credit. One four by six inch note card is permitted.
Good Luck!!!

1. Use the Division Algorithm to prove that all primes greater than 3 must be of the form $6k-1$ or $6k+1$ for some $k \in \mathbb{N}$.

2. Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

3. Prove $(A \cup B) \times C = (A \times C) \cup (B \times C)$.

4. Define a relation $\sim$ on $\mathbb{R}^2$ by stating that $(a, b) \sim (c, d)$ if and only if $a^2 + b^2 \leq c^2 + d^2$. If the relation is an equivalence relation, describe the corresponding partition of $\mathbb{R}^2$. If the relation is not an equivalence relation, state why it fails to be one.

5. Let $f : X \to Y$ be a map with $A_1, A_2 \subset X$. Prove $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$.

6. Prove that

$$1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

for $n \in \mathbb{N}$.

7. Let X be a set. Define the *power set* of X, denoted $\mathcal{P}(X)$, to be the set of all subsets of X. For example,

$$\mathcal{P}(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}.$$

If $X = \{1, 2, \ldots, n\}$ show that $\mathcal{P}(X)$ contains exactly 2^n elements.

8. Prove that there are an infinite number of primes of the form $4m - 1$, where $m \in \mathbb{N}$. *Hint*: Assume there are only a finite number $p_1, p_2, \ldots, p_n$ of primes in the form $4m-1$. Form a number $N = 4p_1p_2 \cdots p_n - 1$.