Name:______________________________

Math S–101. Final Exam—Tuesday, August 15, 2006

Problem Number	Possible Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
Total	50	

Please Read. You have three hours to take this exam. Do any five of the following problems. Please be sure to write neatly in the exam booklet—illegible answers will receive little or no credit. If you do more than five problems, please indicate which problems that you would like us to read by crossing out the remaining solutions. If you fail to do so, only the first five problems in your exam booklet will be read. Any additional problems will be ignored even if they are correct.

Be sure to use correct mathematical notation. To receive full credit on a problem, you will need to justify your answers carefully—unsubstantiated answers will receive little or no credit. You make bring two 8–1/2 by 11 inch sheets of notes.
Good Luck!!!

1. Let $f : X \to Y$ be a well-defined map. Let A and B be subsets of Y. Prove that $f^{-1}(A - B) = f^{-1}(A) - f^{-1}(B)$.

2. Let X and Y be topological spaces and $f : X \to Y$ be a well-defined map. Prove that the following two statements are equivalent.

   (a) For each open set $U \subset Y$, the set $f^{-1}(U)$ is open in X.

   (b) For each closed set $E \subset Y$, the set $f^{-1}(E)$ is closed in X.

   Hint: You may assume Problem 1 is true in your proof.

3. Let $(X, \mathbf{K}_X)$ and $(Y, \mathbf{K}_Y)$ be topological spaces. Suppose that X is connected and $f : X \to Y$ is continuous with $f(X) = Y$. Prove that Y is connected. *Hint*: Assume that Y is not connected.

4. If $f : \mathbb{R}^n \to \mathbb{R}$ is continuous and $c \in \mathbb{R}$, prove that the function $c \cdot f$ defines a continuous function from $\mathbb{R}^n$ to $\mathbb{R}$. Be sure to consider the case $c = 0$.

5. Let $X = \mathbb{R}$ with the Euclidean closure operator $\mathbf{K}$. Prove that the closure of the interval (a, ∞) is $[a, \infty)$.

6. Prove that $\sqrt{6}$ is irrational.

7. Let $(X, \mathbf{K}_X)$, $(Y, \mathbf{K}_Y)$, and $(Z, \mathbf{K}_Z)$ be topological spaces. Suppose that $f : X \to Y$ and $g : Y \to Z$ are continuous. Prove that the composition of f and g,

$$g \circ f : X \to Z,$$

must also be continuous.

8. If two topological spaces $(X, \mathbf{K}_X)$ and $(Y, \mathbf{K}_Y)$ are homeomorphic, we write

$$(X, \mathbf{K}_X) \approx (Y, \mathbf{K}_Y).$$

Prove that $\approx$ is an equivalence relation. *Hint*: You may assume Problem 7 is true in your proof.