

Math S-101

Assignment 8:

Tuesday, July 26, 2005 for Thursday, July 28, 2005

1. Read all of Chapter 2 in the Notes.
2. Find a number x such that $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} \leq x$ for all positive integers n .
Prove your assertion using induction. Then write a few sentences discussing these questions: Can you find a smaller x ? The smallest such x ? What does this have to do with Zeno's Paradox?
3. Find a number x such that $9(1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots + \frac{1}{10^n}) \leq x$ for all positive integers n . Prove your assertion using induction. Then write a few sentences discussing these questions: Can you find a smaller x ? The smallest such x ? What does this have to do with assigning a decimal to the repeating decimal $9.999\bar{9}$?
4. Write out your own proof that the set $A = \{(m, n) \mid m \in \mathbb{Z}^+, n \in \mathbb{Z}^+\}$ consisting of all ordered pairs of positive integers really is countable.
5. Does the union of two countable sets A_1 and A_2 have to be countable? Is the union of a family of countable sets always countable? Care to make a conjecture?
6. Write out your own proof that the power set 2^A is uncountable if $\mathbb{Z}^+ \subset A$.