The following is intended to be an example of a well-written proof by mathematical induction. Remember when I am grading, I am not looking to see if your proof exactly copies this syntax. Rather, I am looking to see if you completely understand what you are doing. Once you have shown me that you understand what induction entails, you will have considerably more freedom to skip non-essential steps in order to make your proof more concise. Accordingly, as the course progresses, a "good" induction proof will look less and less like this example, and proofs you will find in a text will almost never contain this much detail. Nonetheless, this is what I would like to see for your first few assignments.

Theorem 1. *The sum of the first n integer squares is expressible as follows:*

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Proof. We prove this with induction on the integer n. When $n = 1$, $1^2 = \frac{1(1+1)(2*1+1)}{6}$ so this is true. For the induction step, we assume that the statement holds for some $k \in \mathbb{N}$, i.e., that

$$\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$$

for some k. We want to show that this formula holds for $k + 1$. Clearly

$$\sum_{i=1}^{k+1} i^2 = \left(\sum_{i=1}^{k} i^2\right) + (k+1)^2.$$

By our inductive hypothesis,

$$\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6},$$

so we substitute to see that

$$\begin{aligned}
\sum_{i=1}^{k+1} i^2 &= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \\
&= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6} \\
&= \frac{(k+1)[k(2k+1) + 6(k+1)]}{6} \\
&= \frac{(k+1)(2k^2 + 7k + 6)}{6} \\
&= \frac{(k+1)(k+2)(2k+3)}{6} \\
&= \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}
\end{aligned}$$

This last statement is what we wanted to show. Thus, by the Principle of Mathematical Induction, we have shown that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

for all $n \in \mathbb{N}$, and our proof is complete. □