3

b. There is no largest integer. $\forall m \exists n(n > m)$

6

c. $\exists x \forall y(xy = y)$ There exists a multiplicative identity.

7

c. x is y's grandmother. $P(x, z) \wedge P(z, y) \wedge W(x)$

5

b. The statement $\forall y \exists x(x^2 = y)$ is false. This states that for every real y there exists a *real* number x that is its square root. This is clearly false when $y = -1$. Note, this is one example of why it is important to know what the domain is for the variables x and y. If x and y were interpreted to be in $\mathcal{C}$, not $\mathbb{R}$, this statement would be true.

c. The statement $\exists x \forall y(x + 5 = y)$ is false. In English, the reads: "there exists x such that for all y, $x + 5 = y$. But once we have selected x, $x + 5$ is a fixed value, so this clearly can't be true for all $y \in \mathbb{R}$.

7

a. The statement $\exists x P(x) \rightarrow \forall x P(x)$ is false. Let $P(x)$ be the statement x equals -1. There exists $x \in \mathbb{R}$ such that this is true, but it is certainly not true for all $x \in \mathbb{R}$.

b. The statement $[\exists x \forall y P(x, y)] \rightarrow [\forall y \exists x P(x, y)]$ is true. Assume the left hand side is true and let $y \in \mathbb{R}$. We must show that there exists $x \in \mathbb{R}$ such that $P(x, y)$ is true. But by assumption, there is some $x \in \mathbb{R}$ such that $P(x, y)$ is true for all $y \in \mathbb{R}$. So certainly this must be true for our particular y. Thus this is a law of logic.

c. The statement $[\forall y \exists x P(x, y)] \rightarrow [\exists x \forall y P(x, y)]$ is false. Let $P(x, y)$ be the statement $x+y = 0$. Then for all $y \in \mathbb{R}$, there is some x (namely $-y$) such that $x + y = 0$. However, there is no single x such that this is true for all $y \in \mathbb{R}$.