

Homework #3

Due Thursday, October 12, in class

Problem 1 Let A be a set with n elements.

- a) How many functions $f : A \rightarrow A$ are there?
- b) How many of them are injective?

Problem 2 Let A be a set. What functions $f : A \rightarrow A$ are also equivalence relations?

Problem 3 Let A, B, C be sets, $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions.

- a) Prove that if $g \circ f$ is injective then both f and g are injective.
- b) Discuss the assertion " $g \circ f$ is surjective iff f and g are surjective".
- c) Suppose $h : B \rightarrow C$ is a function and $h \circ f = g \circ f$. Show that if f is surjective then $g = h$.

Problem 4 Let A be a set. List all the elements of $A^\emptyset, \emptyset^\emptyset, \emptyset^A$.

Problem 5 Define $f : \mathbb{Z} \rightarrow \mathbb{N}$ by $f(x) = -2x$ for $x \leq 0$ and $f(x) = 2x - 1$ for $x > 0$. Prove that f is a bijection.

Problem 6 Describe the sets $\cup \mathbb{N}$ and $\cap \mathbb{N}$.

Problem 7 Which set has more elements: \mathbb{N}^2 or $2^{\mathbb{N}}$?