Another derivation of Euler's integral formula $n! = \int_0^\infty x^n e^{-x}\,dx$

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Euler's integral formula $\int_0^\infty x^n e^{-x}\,dx$ for $n!$ is usually proved by repeated integration by parts. Inna Zakharevich, a Math 55a student, didn't know the formula and took a few minutes to find the following alternative derivation. Begin with the familiar definite integral

$$\int_0^\infty e^{-ax}\,dx = \left[-\frac{e^{-ax}}{a}\right]_{x=0}^\infty = \frac{1}{a}\,,$$

valid for all $a > 0$. Differentiate n times with respect to a to obtain

$$\int_0^\infty (-x)^n e^{-ax}\,dx = (-1)^n \frac{n!}{a^{n+1}}\,.$$

Now simply take $a = 1$!