Lecture 23

Linearization

Chain rule

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Magic root

$$\sqrt{101} \sim 10 + \frac{1}{20} = 10.04$$

$$\sqrt{101} = 19.0499\ldots$$

Cube roots!

Linearization

$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

Chain rule 1D

$$\frac{d}{dt} f(g(t)) = f'(g(t))g'(t)$$

Example:

$$\frac{d}{dt}\sin(e^{x^2}) = \cos(e^{x^2})e^{x^2}2x$$

Gradient

$$\nabla f(x, y) = \langle f_x, f_y \rangle$$

Chain rule 2D

$$\frac{d}{dt} f(\vec{r}(t)) = \nabla f(\vec{r}(t)) \cdot \vec{r}\,'(t)$$

Related rates

One rule

1

$$f(x, y) = x + y$$

$$\vec{r}(t) = \langle x(t), y(t) \rangle$$

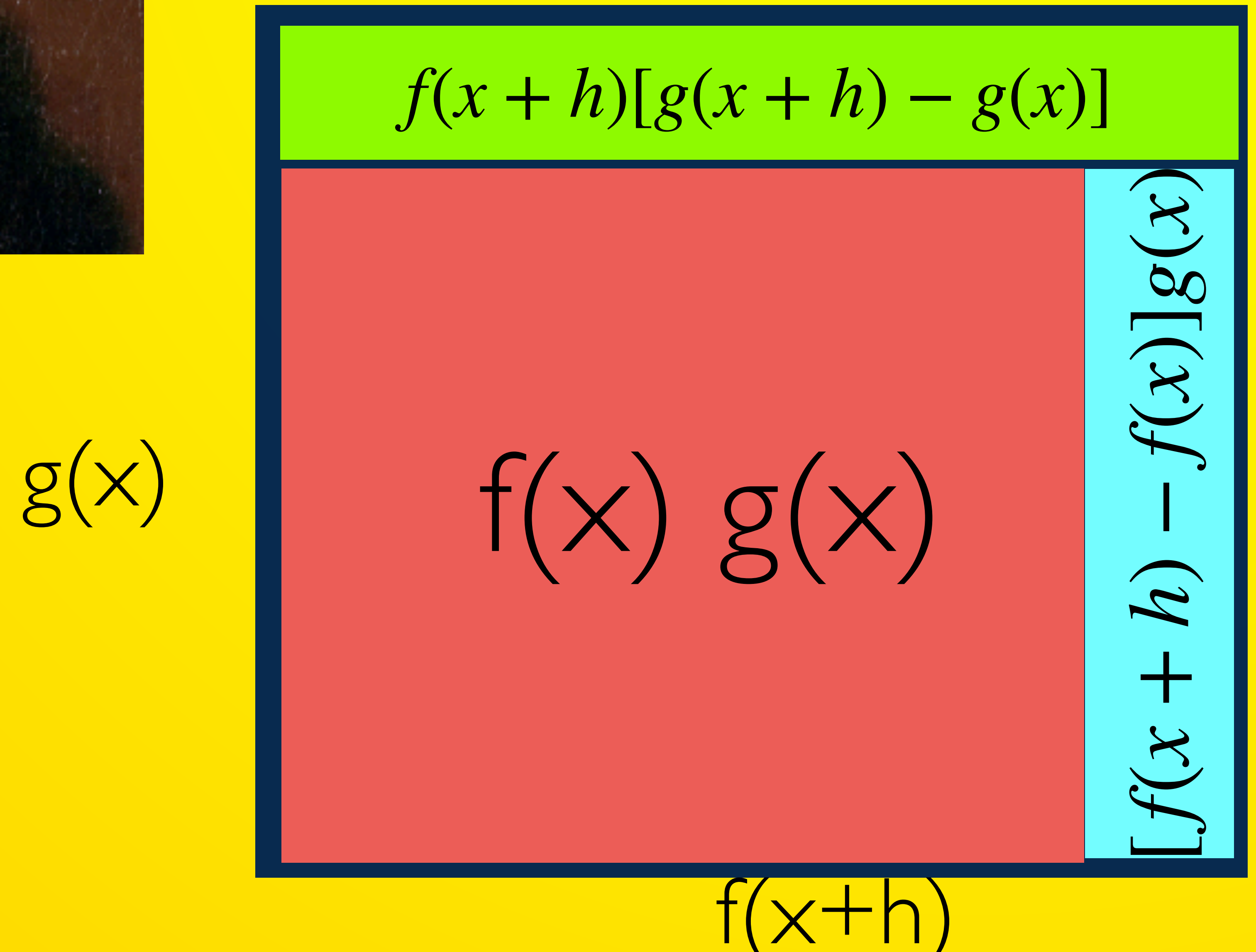
$$(xy)' = \frac{d}{dt} f(\vec{r}(t)) = x' + y'$$

PRODUCT RULE

1646-1716

$$(fg)' = f'g + fg'$$

$$\frac{f(x+h)g(x+h) - f(x)g(x)}{h} = \frac{[f(x+h) - f(x)]g(x)}{h} + \frac{f(x+h)[g(x+h) - g(x)]}{h}$$



Now divide by h

Taking limits $h \to 0$ gives the product rule

SOURCE

Leibniz found it 1675-

...natæ, ad axem normales, VX, WX, YX, ZX, quæ vocentur respective, v, w, y, z; & ipsa AX abscissa ab axe, vocetur x. Tangentes sint VB, WC, YD, ZE axi occurrentes respective in punctis B, C, D, E. Jam recta aliqua pro arbitrio assumta vocetur dx, & recta quæ sit ad dx, ut v (vel w, vel y, vel z) est ad VB (vel WC, vel YD, vel ZE) vocetur dv (vel dw, vel dy vel dz) sive differentia ipsarum v (vel ipsarum w, aut y, aut z) His positis calculi regulæ erunt tales:

Sit a quantitas data constans, erit da æqualis 0, & d $\overline{ax}$ erit æqu. a dx: si sit y æqu v (seu ordinata quævis curvæ YY, æqualis cuivis ordinatæ respondenti curvæ VV) erit dy æqu. dv. Jam *Additio & Subtractio*: si sit $z - y + w + x$ æqu. v, erit d $\overline{z - y + w + x}$ seu dv, æqu $dz - dy + dw + dx$. *Multiplicatio*, d $\overline{xv}$ æqu. $x\,dv + v\,dx$, seu posito y æqu. xv, fiet dy æqu $x\,dv + v\,dx$. In arbitrio enim est vel formulam, ut xv, vel compendio pro ea literam, ut y, adhibere. Notandum & x & dx eodem modo in hoc calculo tractari, ut y & dy, vel aliam literam indeterminatam cum sua differentiali. Notandum etiam non dari semper regressum a differentiali Æquatione, nisi cum quadam cautione, de quo alibi. Porro *Divisio*, d $\frac{v}{y}$ vel (posito z æqu. $\frac{v}{y}$) dz æqu. $\frac{\pm v\,dy \mp y\,dv}{yy}$

Quoad *Signa* hoc probe notandum, cum in calculo pro litera substituitur simpliciter ejus differentialis, servari quidem eadem signa,

$$f(x, y) = xy$$

2

$$\vec{r}(t) = \langle x(t), y(t) \rangle$$

$$(xy)' = \frac{d}{dt} f(\vec{r}(t)) = yx' + xy'$$

$$f(x, y) = x/y$$

3

$$\vec{r}(t) = \langle x(t), y(t) \rangle$$

$$\left(\frac{x}{y}\right)' = \frac{d}{dt} f(\vec{r}(t)) = \left\langle \frac{1}{y}, -\frac{x}{y^2} \right\rangle \cdot \langle x', y' \rangle$$

$$= \frac{x'}{y} - \frac{y'x}{y^2} \qquad = \frac{yx' - xy'}{y^2}$$

QUOTIENT RULE

$$(f/g)' = (f'g - fg')/g^2$$

1646-1716

The product rule gives

$$f' = (g\frac{f}{g})' = g'\frac{f}{g} + g(\frac{f}{g})'$$

$$f'g = g'f + g^2(\frac{f}{g})'$$

Now solve for (f/g)'

$dz - dy + dw + dx$. *Multiplicatio*, dxv æqu. $xdv + vdx$, seu posito y æqu. xv, fiet dy æqu $xdv + vdx$. In arbitrio enim est vel formulam, ut xv, vel compendio pro ea literam, ut y, adhibere. Notandum & x & dx eodem modo in hoc calculo tractari, ut y & dy, vel aliam literam indeterminatam cum sua differentiali. Notandum etiam non dari semper regressum a differentiali Æquatione, nisi cum quadam cautione, de quo alibi. Porro *Divisio*, $d\frac{v}{y}$ vel (posito z æqu. $\frac{v}{y}$) dz æqu. $\frac{\pm v\,dy \mp y\,dv}{yy}$

Quoad *Signa* hoc probe notandum, cum in calculo pro litera substituitur simpliciter ejus differentialis, servari quidem eadem signa, & pro $+z$ scribi $+dz$, pro $-z$ scribi $-dz$, ut ex additione & subtractione paulo ante posita apparet; sed quando ad exegesin valorum venitur, seu cum consideratur ipsius z relatio ad x, tunc apparere, an valor ipsius dz sit quantitas affirmativa, an nihilo minor seu negativa: quod posterius cum fit, tunc tangens ZE ducitur a puncto Z non versus A, sed in partes contrarias seu infra X id est tunc cum infra ordinatam

The Song

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$$

Low Di High
Take High Di Low
Cross the line
and square the low

See you at Halloween!


10-31-2016 Mon 07:07:29
NEXT MONDAY:


ON THE SHOULDERS
PHYSICS
CALCULUS


CRYPTOGRAPHY
ON THE SHOULDERS OF GIANTS
Artificial Intelligence
A Modern Approach
LINEAR ALGEBRA
PHYSICS
Calculus
CALCULUS
APPLIED


Linux Kernel v2.4.0


Linux Kernel v2.4.0


DAY TO DAY SYLLABUS
CALCULUS
CALENDAR


has but
area
finite volume

THE END