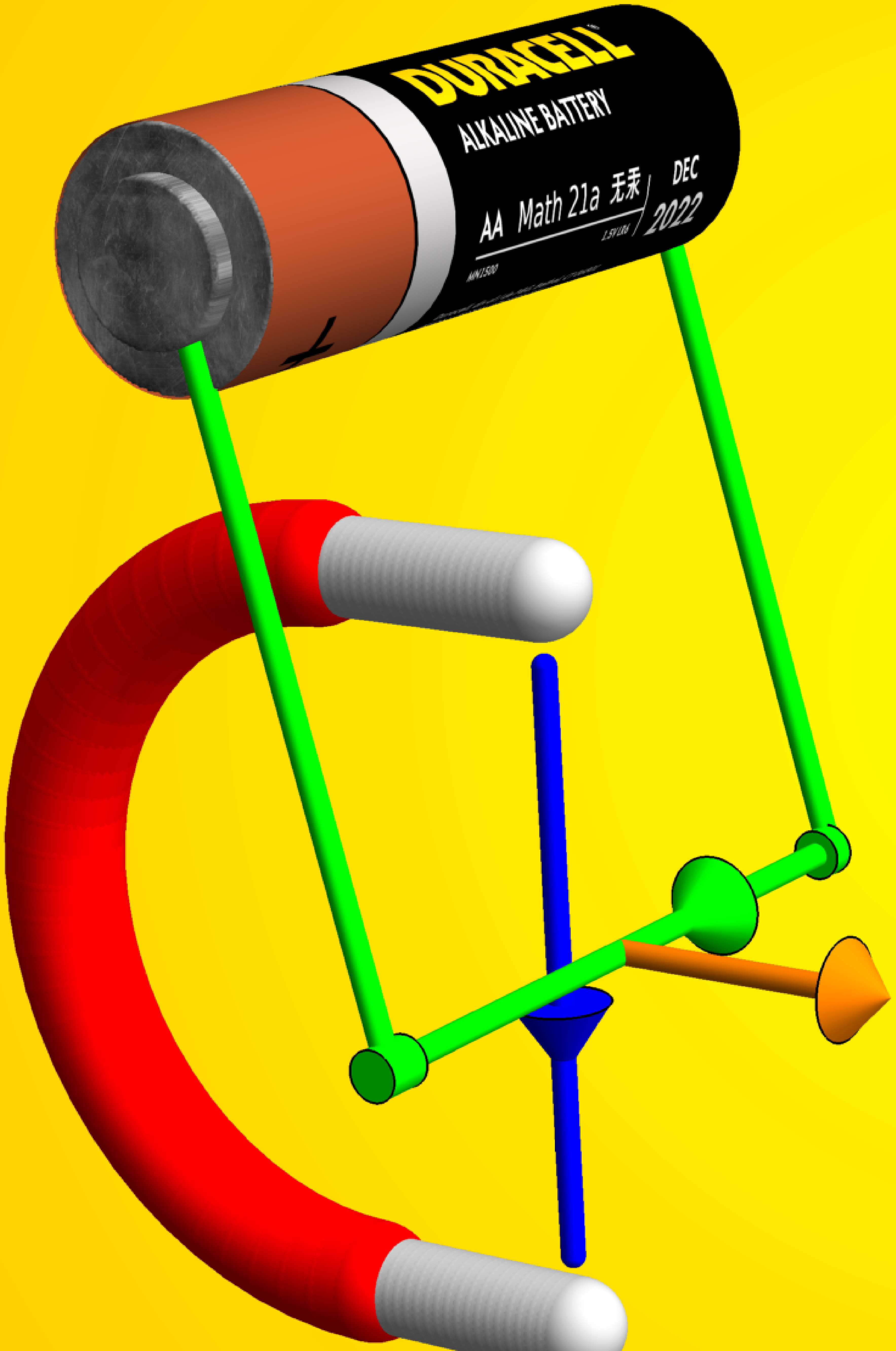


Lecture 13

Cross Product

Table of Contents

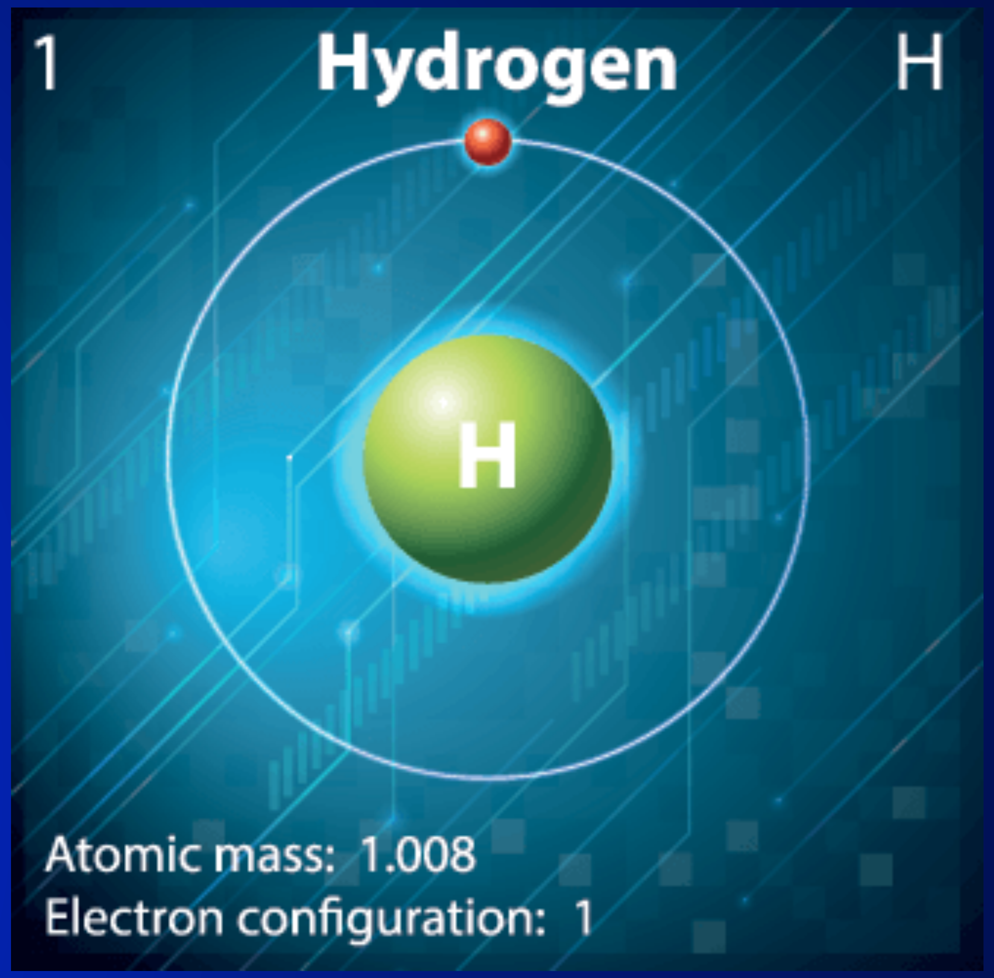
Dot Product Review

Angles

$$\vec{v} \cdot \vec{w} = \|\vec{v}\| \, \|\vec{w}\| \cos(\alpha)$$

$$\cos(\alpha) = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|}$$

Gravity and Quantum



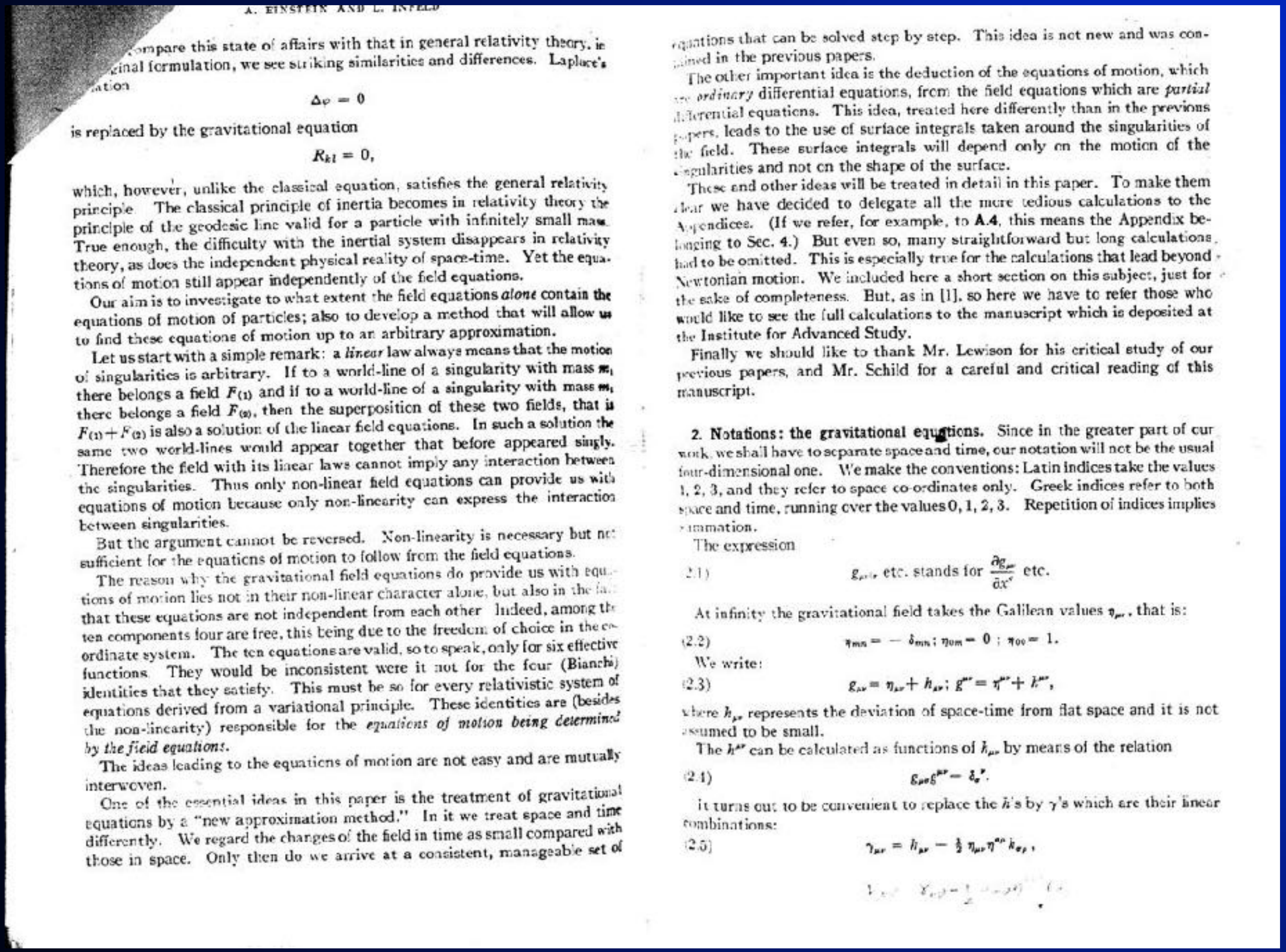
div(F) = ρ

ihψ' =Eψ

Einstein Notation

$$v_i w^i$$

$$A^j_i x^i = b^j$$



…ompare this state of affairs with that in general relativity theory, in …ginal formulation, we see striking similarities and differences. Laplace's …ation

$$\Delta\varphi = 0$$

is replaced by the gravitational equation

$$R_{kl} = 0,$$

which, however, unlike the classical equation, satisfies the general relativity principle. The classical principle of inertia becomes in relativity theory the principle of the geodesic line valid for a particle with infinitely small mass. True enough, the difficulty with the inertial system disappears in relativity theory, as does the independent physical reality of space-time. Yet the equations of motion still appear independently of the field equations.

Our aim is to investigate to what extent the field equations *alone* contain the equations of motion of particles; also to develop a method that will allow us to find these equations of motion up to an arbitrary approximation.

Let us start with a simple remark: a *linear* law always means that the motion of singularities is arbitrary. If to a world-line of a singularity with mass m_1 there belongs a field $F_{(1)}$ and if to a world-line of a singularity with mass m_2 there belongs a field $F_{(2)}$, then the superposition of these two fields, that is $F_{(1)}+F_{(2)}$ is also a solution of the linear field equations. In such a solution the same two world-lines would appear together that before appeared singly. Therefore the field with its linear laws cannot imply any interaction between the singularities. Thus only non-linear field equations can provide us with equations of motion because only non-linearity can express the interaction between singularities.

But the argument cannot be reversed. Non-linearity is necessary but not sufficient for the equations of motion to follow from the field equations.

The reason why the gravitational field equations do provide us with equations of motion lies not in their non-linear character alone, but also in the fa… that these equations are not independent from each other. Indeed, among th… ten components four are free, this being due to the freedom of choice in the co-ordinate system. The ten equations are valid, so to speak, only for six effective functions. They would be inconsistent were it not for the four (Bianchi) identities that they satisfy. This must be so for every relativistic system of equations derived from a variational principle. These identities are (besides the non-linearity) responsible for the *equations of motion being determined by the field equations.*

The ideas leading to the equations of motion are not easy and are mutually interwoven.

One of the essential ideas in this paper is the treatment of gravitational equations by a "new approximation method." In it we treat space and time differently. We regard the changes of the field in time as small compared with those in space. Only then do we arrive at a consistent, manageable set of equations that can be solved step by step. This idea is not new and was con…ained in the previous papers.

The other important idea is the deduction of the equations of motion, which …e *ordinary* differential equations, from the field equations which are *partial* differential equations. This idea, treated here differently than in the previous papers, leads to the use of surface integrals taken around the singularities of the field. These surface integrals will depend only on the motion of the singularities and not on the shape of the surface.

These and other ideas will be treated in detail in this paper. To make them clear we have decided to delegate all the more tedious calculations to the Appendices. (If we refer, for example, to **A.4**, this means the Appendix belonging to Sec. 4.) But even so, many straightforward but long calculations had to be omitted. This is especially true for the calculations that lead beyond Newtonian motion. We included here a short section on this subject, just for the sake of completeness. But, as in [1], so here we have to refer those who would like to see the full calculations to the manuscript which is deposited at the Institute for Advanced Study.

Finally we should like to thank Mr. Lewison for his critical study of our previous papers, and Mr. Schild for a careful and critical reading of this manuscript.

2. Notations: the gravitational equations. Since in the greater part of our work we shall have to separate space and time, our notation will not be the usual four-dimensional one. We make the conventions: Latin indices take the values 1, 2, 3, and they refer to space co-ordinates only. Greek indices refer to both space and time, running over the values 0, 1, 2, 3. Repetition of indices implies summation.

The expression

$$g_{\mu\nu,\sigma} \text{ etc. stands for } \frac{\partial g_{\mu\nu}}{\partial x^\sigma} \text{ etc.} \tag{2.1}$$

At infinity the gravitational field takes the Galilean values $\eta_{\mu\nu}$, that is:

$$\eta_{mn} = -\delta_{mn};\ \eta_{0m} = 0;\ \eta_{00} = 1. \tag{2.2}$$

We write:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu};\ g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}, \tag{2.3}$$

where $h_{\mu\nu}$ represents the deviation of space-time from flat space and it is not assumed to be small.

The $h^{\mu\nu}$ can be calculated as functions of $h_{\mu\nu}$ by means of the relation

$$g_{\mu\sigma}g^{\mu\nu} = \delta_\sigma^\nu. \tag{2.4}$$

It turns out to be convenient to replace the h's by γ's which are their linear combinations:

$$\gamma_{\mu\nu} = h_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}\eta^{\alpha\rho}h_{\alpha\rho}, \tag{2.5}$$

Einstein and Rheinfalls

Dirac Notation

$$\langle \psi | \phi \rangle$$

BRA KET

$$| \phi \rangle \langle \psi |$$


Quantum Gravity

Cross Product

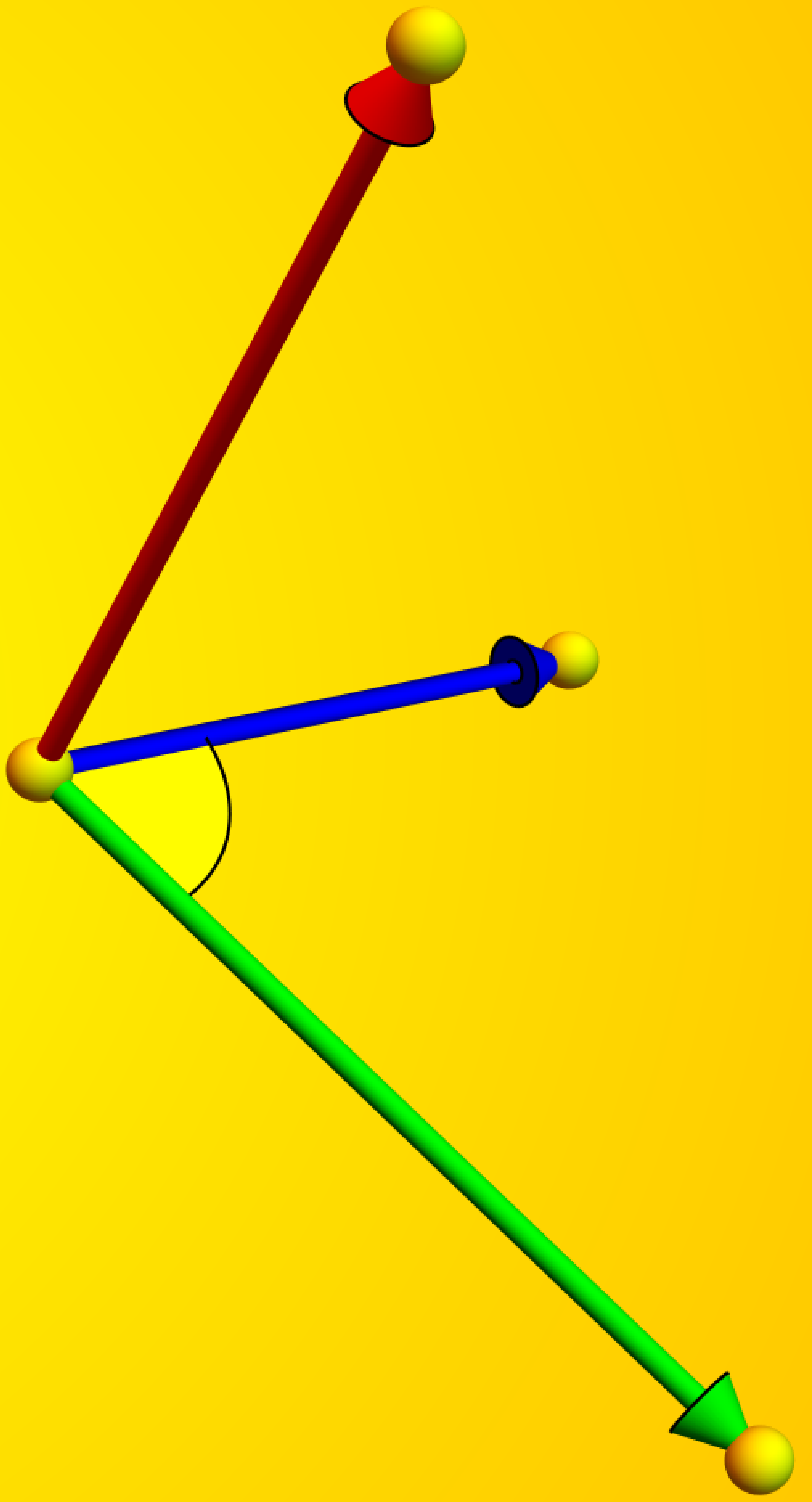
$$\langle 3,4,1 \rangle \times \langle 1,2,1 \rangle = ?$$

$$\langle 3, 4, 1 \rangle$$
$$\langle 1, 2, 1 \rangle$$

$$\langle 4-2, 1-3, 6-4 \rangle = ?$$

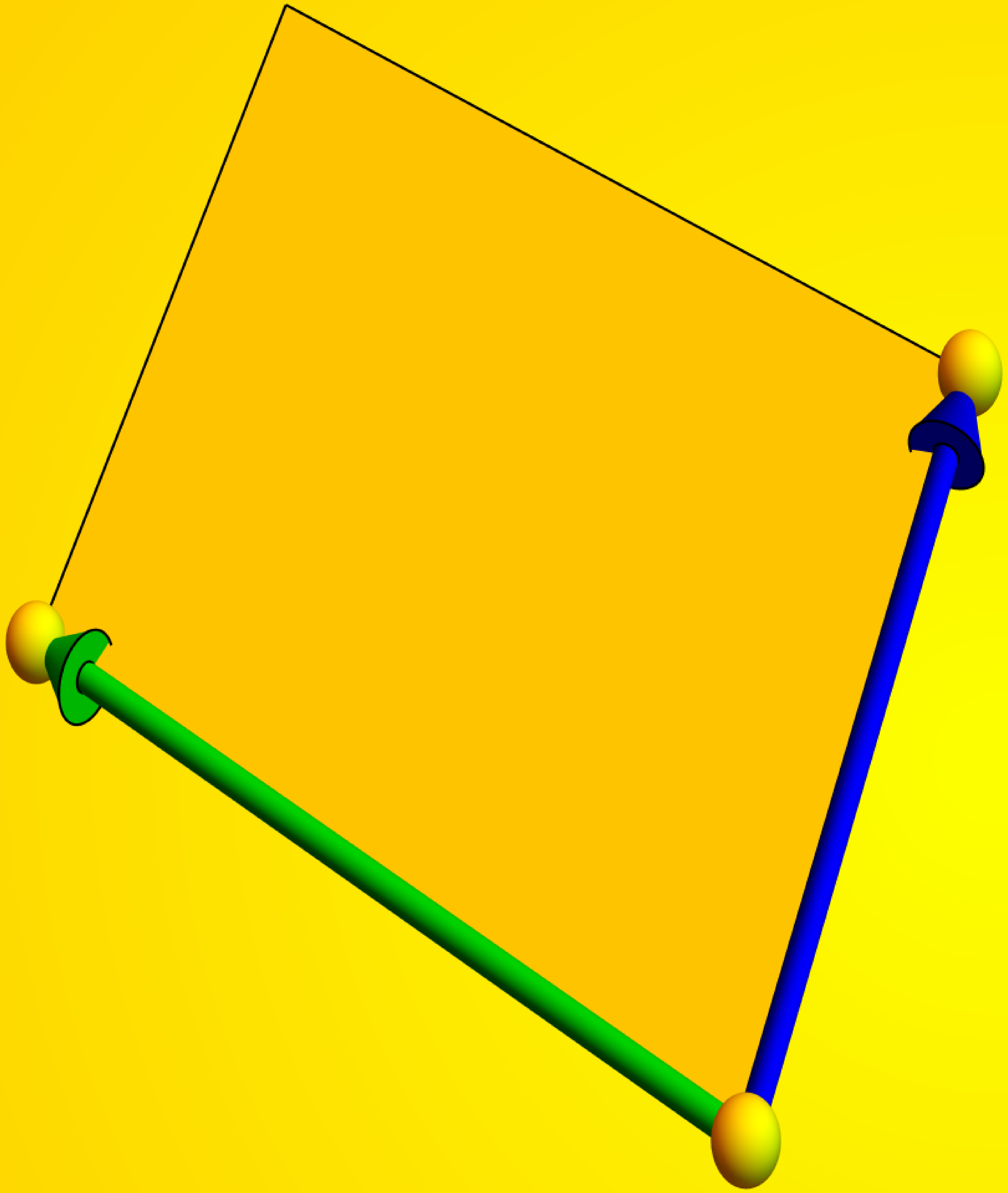
Perpendicular

$$\vec{v} \cdot (\vec{v} \times \vec{w}) = 0$$
$$\vec{w} \cdot (\vec{v} \times \vec{w}) = 0$$



Magnitude

$$|\vec{v} \times \vec{w}| = \|\vec{v}\| \, \|\vec{w}\| \sin(\alpha)$$



Area

base height

$$|\vec{v} \times \vec{w}| = \|\vec{v}\| \, \|\vec{w}\| \sin(\alpha)$$

Cauchy-Binet

$$\|\vec{v} \times \vec{w}\|^2 + |\vec{v} \cdot \vec{w}|^2 = \|\vec{v}\|^2 \|\vec{w}\|^2$$

```
v = {v1, v2, v3}; w = {w1, w2, w3}; u = Cross[v, w];
Simplify[(v . w)^2 + u . u == v . v*w . w]
```

History


HAMILTON
STREAMING EXCLUSIVELY JULY 3
PARENTS STRONGLY CAUTIONED
PG-13
LANGUAGE AND SOME SUGGESTIVE MATERIAL

William Rowan Hamilton

1805-1865

Quaternion Discovery

1843

$$i^2 = j^2 = k^2 = ijk = -1$$


Europower Battery Centre
Speedy Services
Dublin Industrial Es
Colorman (Ireland)
Broom Bridge
Broome Bridge Graffiti Quaternions...
PIRATE.COM - Rehearsal S
Broombridge
Broombridge Luas Depot

Applications

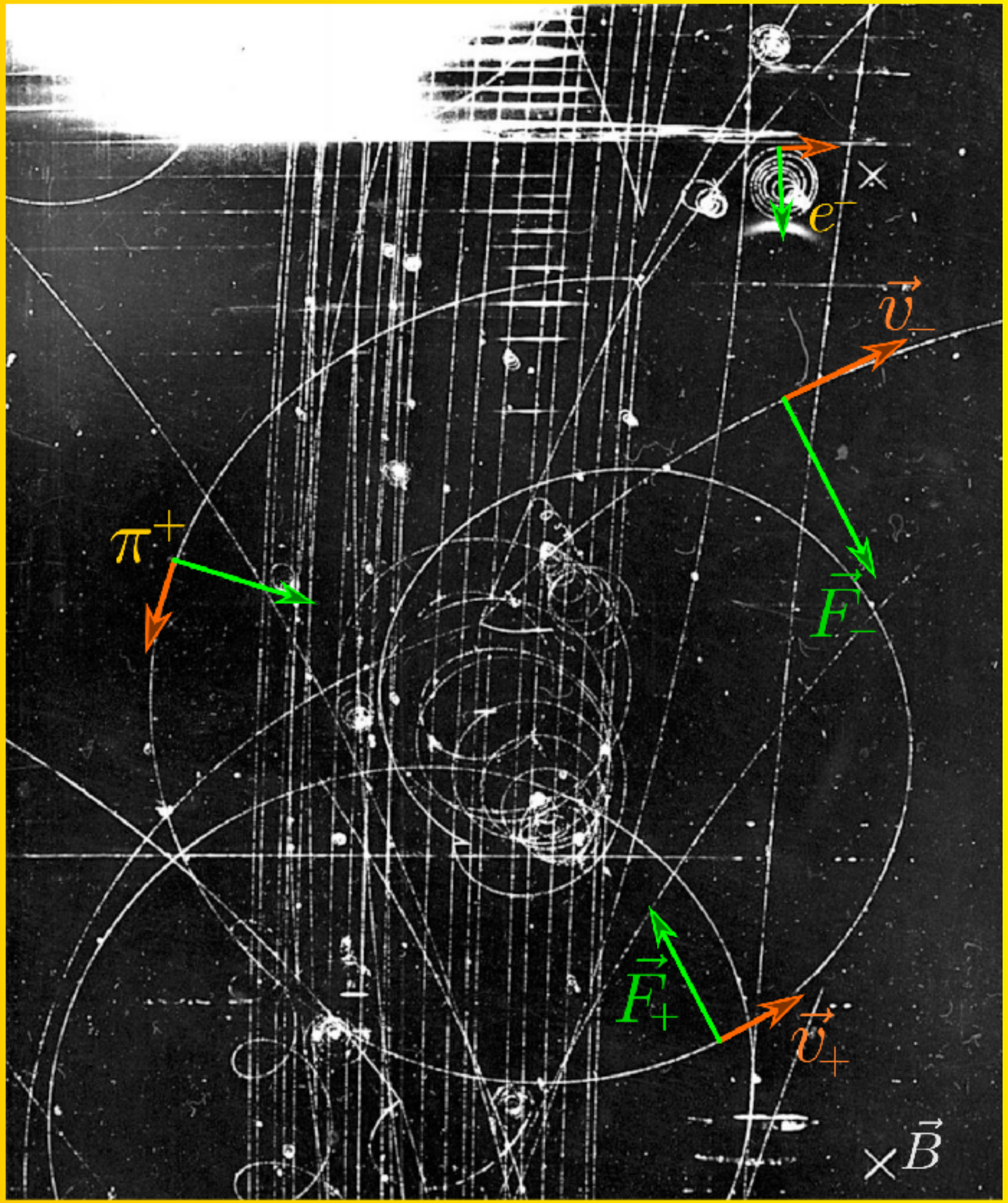
Angular momentum	$mr \times v$
Centrifugal force	$m\omega \times (r \times \omega)$
Coriolis force	$2mv \times \omega$
Torque	$r \times F$
Lorentz force	$qv \times B$

Coriolis Force

Credit: McDougal Littel Exploring Earth visualization

Coriolis Force

Lorentz force

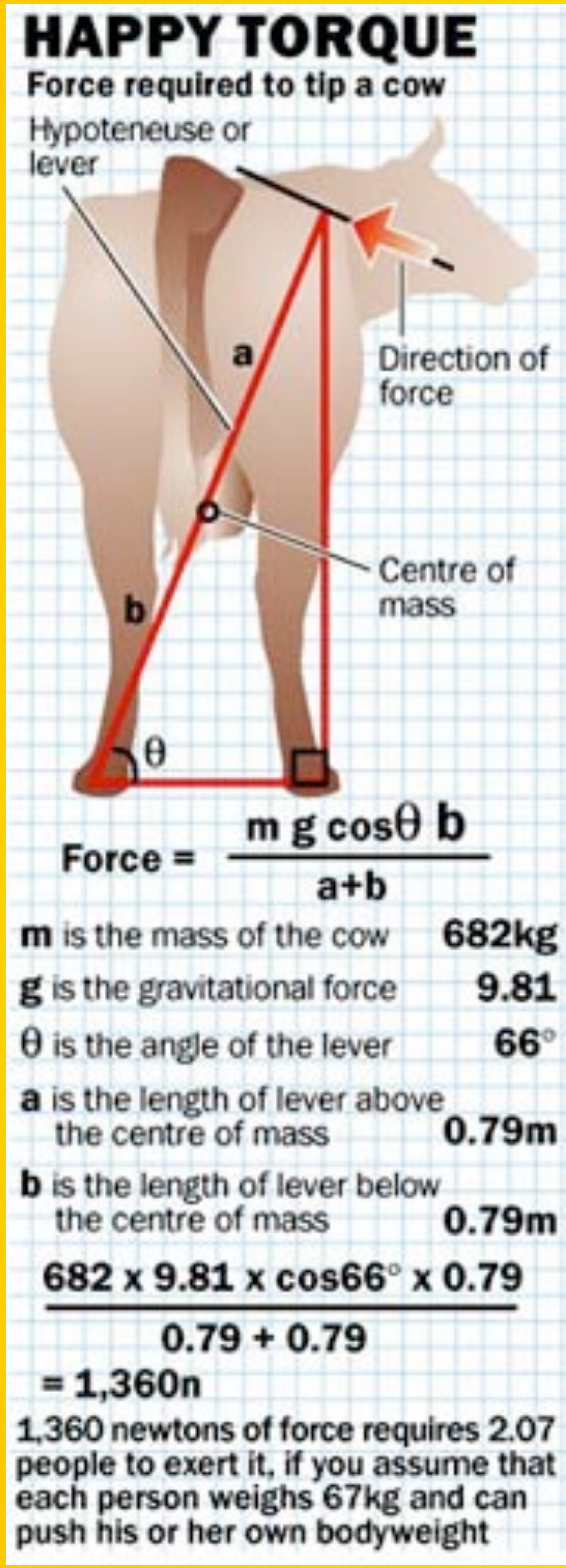


$$qv \times B$$

Lorentz force

$$qv \times B$$

r × F
Torque



Cow Tipping problem

IS IT POSSIBLE FOR A HUMAN TO PUSH OVER A COW?

Cow tipping

From Wikipedia, the free encyclopedia

For the Beavis and Butt-Head *episode, see* *Cow Tipping (*Beavis and Butt-Head *episode)*.

Cow tipping is the purported activity of sneaking up on any unsuspecting or sleeping upright cow and pushing it over for entertainment. The practice of cow tipping is generally considered an urban legend,[1] and stories of such feats viewed as tall tales.[2] The implication that rural citizens seek such entertainment due to lack of alternatives is viewed as a stereotype.[3][4] The concept of cow tipping apparently developed in the 1970s, though tales of animals that cannot rise if they fall has historical antecedents dating to the Roman Empire.

Cows routinely lie down to sleep.

Cows routinely lie down and can easily regain their footing unless sick or injured. Scientific studies have been conducted to determine if cow tipping is theoretically possible, with varying conclusions. All agree that cows are large animals that are difficult to surprise and will generally resist attempts to be tipped. Estimates suggest a force of between 3,000 and 4,000 newtons (670 and 900 pounds-force) is needed, and that at least four and possibly as many as fourteen people would be required to achieve this. In real-life situations where cattle have to be laid on the ground, or "cast", such as for branding, hoof care or veterinary treatment, either rope restraints are required or specialized mechanical equipment is used that confines the cow and then tips it over. On rare occasions, cattle can lie down or fall down in proximity to a ditch or hill that restricts their normal ability to rise without help. Cow tipping has many references in popular culture and is also used as a figure of speech.

Apropos Bulls

Apropos Cauchy-Binet

$$1-\cos^2(x) = \sin^2(x)$$ Pythagoras

$$|v|^2 |w|^2 - (v.w)^2 = |v \times w|^2$$ Lagrange

$$\det \begin{vmatrix} v.v & v.w \\ v.w & w.w \end{vmatrix} = | v \wedge w |^2$$ Cauchy-Binet

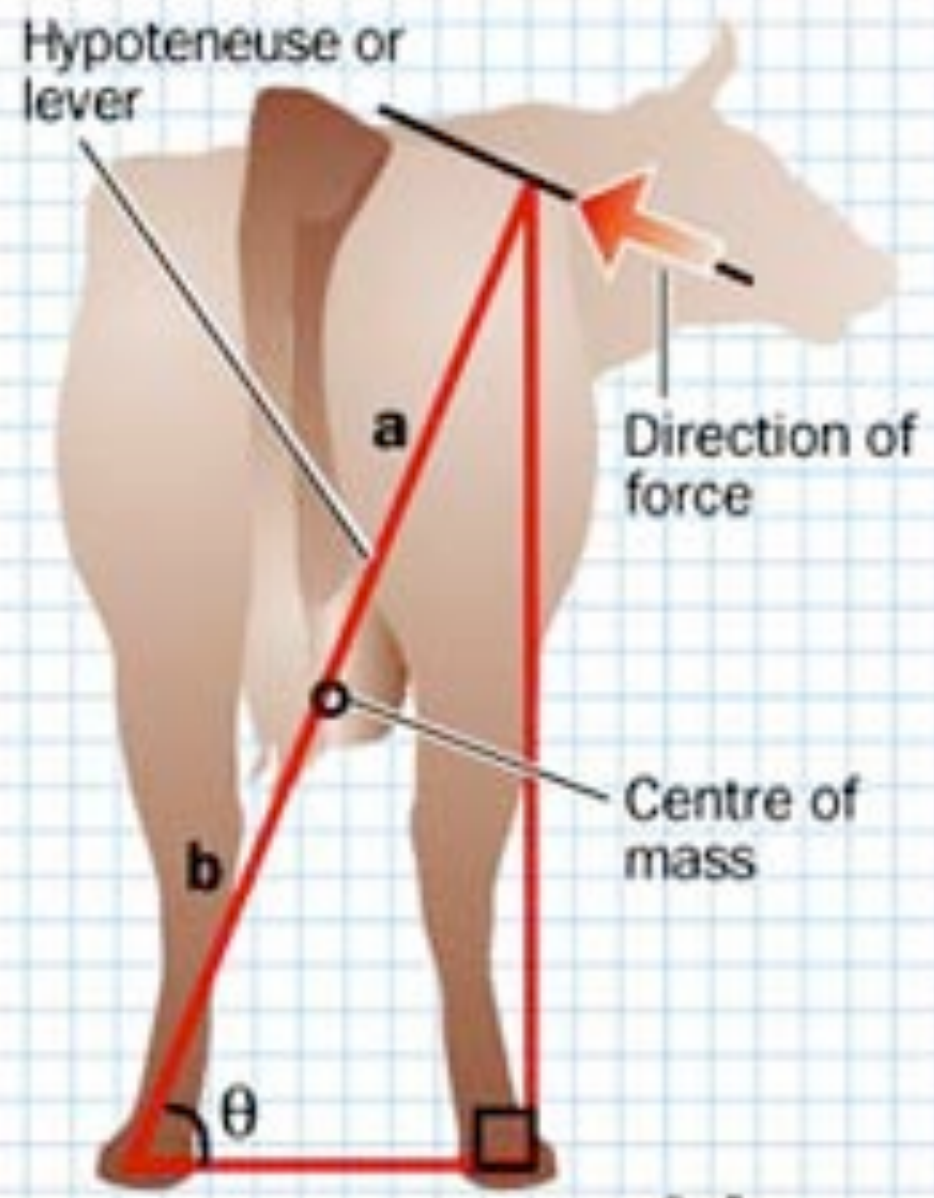
$$\det(1+F^T F) = \sum_P \det(F_P)^2$$ Knill

Cow Tipping problem

Is it possible for a human to push over a cow?

HAPPY TORQUE

Force required to tip a cow



$$\text{Force} = \frac{m\, g \cos\theta\, b}{a+b}$$

m is the mass of the cow	682kg
g is the gravitational force	9.81
θ is the angle of the lever	66°
a is the length of lever above the centre of mass	0.79m
b is the length of lever below the centre of mass	0.79m

$$\frac{682 \times 9.81 \times \cos 66^\circ \times 0.79}{0.79 + 0.79}$$

= 1,360n

1,360 newtons of force requires 2.07 people to exert it, if you assume that each person weighs 67kg and can push his or her own bodyweight

Cow tipping

From Wikipedia, the free encyclopedia

For the Beavis and Butt-Head *episode, see* *Cow Tipping (*Beavis and Butt-Head *episode)*.

Cow tipping is the purported activity of sneaking up on any unsuspecting or sleeping upright cow and pushing it over for entertainment. The practice of cow tipping is generally considered an urban legend,[1] and stories of such feats viewed as tall tales.[2] The implication that rural citizens seek such entertainment due to lack of alternatives is viewed as a stereotype.[3][4] The concept of cow tipping apparently developed in the 1970s, though tales of animals that cannot rise if they fall has historical antecedents dating to the Roman Empire.

Cows routinely lie down to sleep.

Cows routinely lie down and can easily regain their footing unless sick or injured. Scientific studies have been conducted to determine if cow tipping is theoretically possible, with varying conclusions. All agree that cows are large animals that are difficult to surprise and will generally resist attempts to be tipped. Estimates suggest a force of between 3,000 and 4,000 newtons (670 and 900 pounds-force) is needed, and that at least four and possibly as many as fourteen people would be required to achieve this. In real-life situations where cattle have to be laid on the ground, or "cast", such as for branding, hoof care or veterinary treatment, either rope restraints are required or specialized mechanical equipment is used that confines the cow and then tips it over. On rare occasions, cattle can lie down or fall down in proximity to a ditch or hill that restricts their normal ability to rise without help. Cow tipping has many references in popular culture and is also used as a figure of speech.


Lets Try!


But then…

Homework due Wednesday

THE END