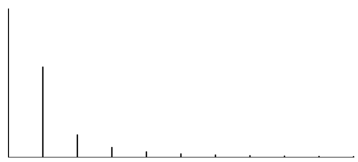
Discrete probability distributions (II)

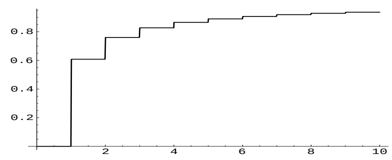
ZETA DISTRIBUTION. $\Omega = \mathbb{N} \setminus \{0\} = \{1, \ldots, \}$.

$$\boxed{\mathrm{P}[\{X(\omega) = k\}] = \zeta(s)^{-1} n^{-s}}$$

For fun. Useful in number theory, where it gives an elegant proof of the Euler formula

$$\zeta(s)^{-1} = \prod_{p \in \mathbb{N}\ p \in \mathbb{N}\ \text{p is a prime}} (1 - p^{-s}) \ .$$

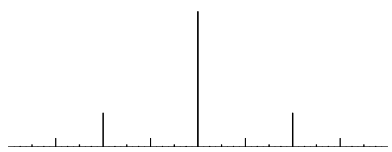


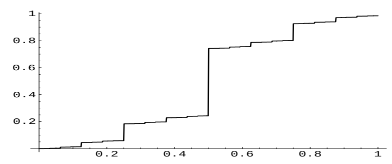


DEVILISH DISTRIBUTION. $\Omega = \{k/2^n, 1 \leq k \leq 2^n, k \text{ odd }\}$.

$$\boxed{\mathrm{P}[\{X(\omega) = k2^{-n}\}] = 2/4^n.}$$

Such distributions occur in quantum mechanics in crystallic alloys or random materials The function F is there called the density of states. The spikes of the density correspond to the location of the possible energy values of the electron.





NEGATIVE BINOMIAL DISTRIBUTION. $\Omega = \{0, 1, \ldots, \}$. parameters $0 \leq p \leq 1$, $n \in \mathbb{N}$.

$$\boxed{\mathrm{P}[\{X(\omega) = k\}] = \begin{pmatrix} n + k - 1 \\ n - 1 \end{pmatrix} p^n (1 - p)^k}$$

Distribution of number of failures that occur prior to the n'th success, when iterating experiments which succeed with probability p.