Math464/564 O. Knill, **March 1997**

Homework 9

Due: Tuesday, April 1, 1997 in class

Topic: Absolutely continuous random variables.

Remark. This HW contains 5 questions. Each question can give 10 points, so that you can get a maximal credit of 50 points for this HW.

1) (10 points) Topic: **Discrete and continuous random variables**.
Decide from each of the following random variables on $[0,1]$ whether it is discrete or absolutely continuous:

a) $X(\omega) = \sin(\omega)$.
b) $X(\omega) = \pi/5$.
c) $X(\omega) = [1000\cdot\omega]$, where $[x]$ is the largest integer smaller or equal to x (for example $[3.1415926...] = 3$).
d) $X(\omega) = 1_{\omega=0.5}$, where $1_A(\omega) = 1$ if $\omega \in A$ and $1_A(\omega) = 0$ if $\omega \notin A$.
e) $X(\omega) = 1_{[0.2,0.3]}(\omega)$.

2) (10 points) Topic: **Normal distribution**. Consider the normal distribution with probability density

$$f(x) = (2\pi\sigma^2)^{-1/2}\, e^{-\frac{(x-m)^2}{2\sigma^2}} \; .$$

a) Show that $f(x)$ is maximal for $x = m$.
b) For $x = m \pm \sigma$, the second derivative of f is vanishing.

3) (10 points) Topic: **The Erlang distribution**. Consider a random variable X with probability density function

$$f(x) = \frac{\lambda^k x^{k-1}}{(k-1)!} e^{-\lambda x} 1_{x \geq 0} \; ,$$

where λ, k are parameters. This is called the Erlang distribution. It arises naturally because as we will see later, the sum of independent exponential distributed random variables is Erlang distributed.

a) Show that the exponential distribution is a special case.
b) Verify that f is indeed a density functions, that means, verify that $\int_0^\infty f(t)\, dt = 1$.
c) Compute $\mathrm{E}[X]$.
d) Compute $\mathrm{Var}[X]$.

4) (10 points) Topic: **The Exponential distribution**. A radioactive sample containing **Lutetium** (Lu) [1] emits α rays (He^4 nuclei). Assume the waiting time X (measured in seconds) for a decay is exponentially distributed with parameter $\lambda = 3$.

a) What is the probability to get a decay in 1 second?
b) How long does one have to wait in average to measure a decay?

5) (10 points) Topic: **General probability spaces**.
It might surprise that there exist open dense sets on the interval $[0,1]$ which have not full probabiliy. Enumate the rational numbers $x_1, x_2, \dots$ and define $A = \bigcup_n \{x \mid |x - x_n| < 4^{-n}\}$. The set A is dense because it contains the rational numbers. The set A is open because it is a union of open intervals. What is the measure of A?

[1] A Mathematica standard package gives more information: <<Miscellaneous ‘ ChemicalElements ‘ AtomicNumber[Lutetium] AtomicWeight[Lutetium]