Math464/564 O. Knill, **March 1997**

Homework 8

Due: Tuesday, March 25, 1997 in class

Topic: Review of the first half of the course.

Remark. This HW contains 5 questions. Each question can give 10 points, so that you can get a maximal credit of 50 points for this HW. This problem set is a review and covers also older topics. The aim is to improve your routine before we go into the next half of the course after the spring break.

1) (10 points) Topic: **Probability space**.

a) Prove $P[A] = P[A \cap B] + P[A \cap B^c]$.
b) Prove $P[A \Delta A] = 0$.
c) Find $P[A \cap B]$ if A and B are independent events of probability $1/4$.
d) Prove $1 = P[B|A] + P[B^c|A]$.
e) Prove the formula $P[A \cap B] \geq P[A] + P[B] - 1$.

2) (10 points) Topic: **Combinatorics**.

a) What is the probability to have 4 kings in a poker hand. (5 cards out of 52)?
b) A family has three childs. What is the probability that two of the childs are boys?
c) A Morse code consists of a sequence of dots and dashes. A letter is encoded by a sequence of dots and dashes with length 1 to 5. (Repetitions are allowed). How many letters can be encoded with 1 to 5 symbols?

(See http://www.soton.ac.uk/ scp93ch/refer/alphabet.html for the Morse code alphabet on the web.)

3) (10 points) Topic: **Probability distributions** Let X and Y be independent random variables with Poisson distribution $\lambda = 1$ and $\mu = 5$.

a) Find $P[X \geq Y]$.
b) Find $P[X = Y]$.

4) (10 points) Topic: **Expectation, Variance etc.** .

a) Compute the probability generating function ϕ_X of a $\mathbb{N}$-valued random variable X with probability density function $f_X(n) = 2^{-n}$, $n = 1, 2, \dots$. (there should be no sum in the end).
b) Use a) to compute $E[X]$.
c) Use a) to compute $\mathrm{Var}[X]$.
d) Assume X and Y are independent. We know $E[X] = 2, E[Y] = 3, \mathrm{Var}[X] = 4, \mathrm{Var}[Y] = 2$. Find $\mathrm{Var}[7X - 2Y]$.
e) Let X,Y be as in d). Find $\rho[X + Y, Y] = \mathrm{Corr}[X + Y, Y]$.

5) (10 points) Topic: **Chebychev-Markov inequality**. Prove Cantelli's inequality

$$P[|X - E[X]| \geq \epsilon] \leq \frac{2\mathrm{Var}[X]}{\epsilon^2 + \mathrm{Var}[X]} \, .$$

$\boxed{\text{Have a relaxing spring break!}}$