Math464/564 O. Knill, **February 1997**

Homework 5

Due: Tuesday, February 25, 1997 in class

Topics: Discrete random variables, independence of random variables, discrete random vectors.

Remark. This HW contains 3 questions. The number of questions is reduced because of possible midterm stress this week. You can here get a maximal credit of 30 points.

1) (10 points) Topic: **Independent random variables** . What is the distribution of the sum of two independent random variables X, Y where X is Poisson distributed with parameter $\lambda = 2$ and Y is Poisson distributed with parameter $\lambda = 3$?

2) (10 points) Topic: **Poisson approximation of Bernoulli**. The production of microchips becomes more and more delicate. What is the probability that 2 chips of a silicon waffer containing 1000 chips are defect, if a chip is defect with probability 0.01. Approximate the Bernoulli distribution by a suitable Poisson distribution.

3) (10 points) Topic: **Random vectors, multinomial distribution**. If $X = (X_1, \dots, X_r)$ is a random vector, then the distribution of X_i is called a marginal distribution of X. What is the marginal distribution of X_1 of the random vector $X = (X_1, X_2, X_3)$, if X has the multinomial distribution $\mathrm{P}[X = k] = \binom{10}{k_1, k_2, k_3} 2^{-10}$ if $k_1 + k_2 + k_3 = 10$ and $\mathrm{P}[X = k] = 0$ if $k_1 + k_2 + k_3 \neq 10$.

Checklist for first midterm.

The material is about the topics of the first three chapters of the book which we covered as weill as the material which appeared in class. There will be a short theory part (multiple choice) and some problems in the style of the homework. You will be allowed to use all your notes, all in class distributed material, all homwork, but no printed books (the later restriction is to prevent you from "reading " and losing time during the exam and to encourage to do a private collection of notes. My suggestion for a preparation: look again at the homework problems HW1-HW4 (can I solve all problems now ?), make a short private list of important definitions, formulas, results and examples. Topics,

PROBABILITY SPACES.

- Definition of σ-algebra, probability measure, probability spaces.
- Examples of probability spaces.
- Working in the algebra of sets.
- Basic formulas like $\mathrm{P}[A] = 1 - \mathrm{P}[A^c]$ or the switch on switch off Formula of Sylvester.

COMBINATORICS.

- Knowledge of all basic formulas in combinatorics.
- Some formula deduced from basic formulas like Bernoulli or the formula in lotto.
- Know at least one example for each basic formula.

DISCRETE RANDOM VARIABLES.

- Definition of a discrete random variable.
- Definition of density and distribution function.
- Know from the basic distributions what is set of values, what are the parameters and an example and where it is used.