Math464/564 O. Knill, **January 1997**

Homework 2

Due: Tuesday, February 4, 1997 in class

Topics: Modelling of probability spaces, conditional probability, independence.

Remark. This HW contains 5 questions. Each question can give 10 points, so that you can get a maximal credit of 50 points for this HW.

1) (10 points) Topic: **Modeling probability spaces, a political problem**.
The president of a country comes from a family of two children. What is the probability that the other child is his sister?

2) (10 points) Topic: **Conditional probability spaces, Polya's urn scheme**.
An urn has 16 balls, 12 red and 4 black balls. Without putting the balls back, one draws two balls, one after the other mixing the balls in urn before each drawing. Assume the second ball is red. What is the probability that the first ball is red.

3) (10 points) Topic: **Independence, the complement events**.
Let A and B be two independent events.
a) (5 points) Prove that A and B^c are independent.
b) (5 points) Prove that A^c and B^c are independent.

4) (10 points) Topic: **Independence, overbooking of flights**
Experience shows that 5 % of the people reserving flights do not show up. Airlines therefore decide to overbook the flights and take the risk to pay out people with no seat. Assume a plane has 230 seats and that the airline sells 233 tickets. What is the probability that all people will be accommodated?

5) Topic: (10 points) Topic: **Probability spaces, combinatorics, the wardrobe problem**.
We distribute randomly $n \in \mathbb{N}$ coats of n people and wonder what is the probabilihy that no person gets his or her coat.
a) (2 poits) Set up the probability space $(\Omega, \mathcal{A}, P)$ for the situation.
b) (2 points) Define for $1 \leq i \leq n$ the event A_i that at least person number i gets his (or her) coat. What is the probability of the event $A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_k}$, where $1 \leq i_1 < i_2 < \ldots < i_k \leq n$.
b) (3 points) What is the probability $p(n)$ that no person gets his (or her) own coat? Hint: Consider the event that at least one person gets his (or her) coat and use the formula proven in class

$$P[\bigcup_{i=1}^{n} A_i\,] = \sum_{k=1}^{n}(-1)^{k-1} \sum_{1 \leq i_1 < i_2 < \ldots < i_k \leq n} P[A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_k}]\,.$$

c) (3 points) What happens with the probability $p(n)$ for $n \to \infty$?