Math464/564 O. Knill, **May 1997**

Homework 14

Due: Tuesday, May 6, 1997 in class

Topic: Random walks, gambling

Remark. This HW contains 5 questions with 10 points giving a total of 50 points.

1) (10 points) Topic: **Polya's theorem** .
You have seen that the reason why the random walk in two or less dimensions returns and not in higher dimensions is that

$$\int_{x \in \mathbb{R}^d \mid |x| \leq r} |x|^{-2} \, dx$$

is finite if and only if $d \geq 3$. Verify this fact.

2) (10 points) Topic: **Can you beat the system with insider information?** .
We have proven that for a gambling system V and a fair random walk S_n, one can not beat the system. That is, the expectation for the winning $\mathrm{E}[S_N^V] = 0$. It was assumed that V_k was previsible that is it depends only on $X_1, \dots X_{k-1}$. Assume you drop this information and you allow V_k to depend also on X_k (you have some insider information). Design a concrete gambling system, which provides a win.

3) (10 points) Topic: **Gambling system for random walk with drift**.
Assume you have a random walk S_n with drift, that is X_k are IID random variables with expectation $\mathrm{E}[X_k] = p$. Assume you have a gambling system V_k. Give a formula for the expected win $\mathrm{E}[S_n^V]$ after time n.

4) (10 points) Topic: **Breaking the bank**.
Assume you play in a fair casino which is a random walk with $\mathrm{E}[X_k] = 0$. Assume the bank has $a = 1000$ dollars available and you have $b = 2000$ dollars available. If $S_n = X_1 + \dots X_n = -a$, then you broke the bank, if $S_n = b$, then you lost all your money and you are broke. What is the probability that you break the bank?

5) (10 points) Topic: **The variance of the winning**.
You play the game as in the previous question but this time, the casino is no more fair; $\mathrm{E}[X] = const$. The bank knows empirically $\mathrm{E}[S_T]$, the average win in the game, where the bank has initially a dollars and a player has b dollars and $T = T_{a,b}$ is the ruin time for one of the players. What is the variance of S_T?