Math464/564 O. Knill, **April 1997**

Homework 11

Due: Tuesday, April 15, 1997 in class

Topic: Random vectors, joint distributions, marginal density, distributions of a sum of two independent random variables, transformation of densities, conditional density, Bayes rule.

Remark. This HW contains 5 questions each giving 10 points so that you can get a maximal credit of 50 points for this HW.

1) (10 points) Topic: Marginal density. Let X, Y be absolutely continous with density $f(x,y) = \lambda^2 e^{-\lambda y}$ for $0 \leq x \leq y$ and $f(x,y) = 0$ else.

   a) Sketch the level curves of f.
   b) Find the marginal density of X.
   c) Find the marginal density of Y.

2) (10 points) Topic: Covariance matrix, normal distribution. Assume X and Y are $N(0,1)$-distributed random variables satisfying $\mathrm{Cov}[X,Y] = 2$. Find the density of the random vector $(X+2Y, X+Y)$.

   Hint: you can use the fact that a Gaussian random vector $X = (X_1, \ldots, X_n)$ with covariance matrix $K_{ij} = \mathrm{Cov}(X_i, Y_j)$ and vector mean $m = (m_1, \ldots, m_n)$ with $m_i = E[X_i]$ has the density

$$f(x) = (2\pi)^{-n/2} |\det K|^{-1/2} e^{-\frac{1}{2}(x-m)^T K^{-1}(x-m)} .$$

   Compute first the 2×2 covariance matrix K of $(X+2Y, X+Y)$.

3) (10 points) Topic: Distribution of a sum of independent random variables. Let X, Y be independent random variables both with continuous distribution f. What is the density of the random variable $Z = X - Y$.

4) (10 points) Topic: Transformation of joint densities. Let (X, Y) be a random vector with density $f(x,y) = 6e^{-2x-3y}$. What is the joint density of the random vector $(3X - Y, X)$.

5) (10 points) Topic: Conditional density, Bayes rule.
   Suppose you know that $f_{Y|X}(x|y)$ is $N(0,4)$-distributed (normal distributed with mean $m = 0$ and variance $\sigma^2 = 4$) and that X is $N(0,1)$-distributed. Compute $f_{X|Y}(x|y)$.