Math464/564 O. Knill, **January 1997**

Homework 1

Due: Tuesday, January 28, 1997 in class

Topics:

Remark. This HW contains 5 questions. Each question can give 10 points, so that you can get a maximal credit of 50 points for this HW.

1) (10 points) Topic: **Boolean algebras: a group**. Let $\mathcal{A}$ be a Boolean algebra. Define $A \Delta B = (A \cup B) \setminus (A \cap B)$. Show that $(\mathcal{A}, \Delta)$ is a commutative group (this means the following properties hold:)
a) $A, B \in \mathcal{A} \Rightarrow A \Delta B \in \mathcal{A}$.
b) For all A, B, C the **associativity law** $(A \Delta B) \Delta C = A \Delta (B \Delta C)$ holds.
c) There exists a **zero element** $A_{zero} \in \mathcal{A}$ such that $A \Delta A_{zero} = A$ for all $A \in \mathcal{A}$.
d) For all A, there exists an **inverse** B such that $A \Delta B = A_{zero}$.
e) For all A, B the **commutativity law** holds $A \Delta B = B \Delta A$.

2) (10 points) Topic: **Probability spaces: internet**. Consider a part of the internet computer network consisting of 3 nodes (computers). Assume that two of the three computers are connected with probability p and that with probability $q = 1 - p$, the connection is broken.
a) Construct the probability space $(\Omega, \mathcal{A}, P)$ and find the event that all nodes have connection to all other nodes.
b) What is the probability that one can reach from any of the computers any other computer.

3) (10 points) Topic: **Probability spaces: dices**. Suppose two dice are rolled once and that each of the 36 possible outcomes are equally likely. What is the probability that the product of the two numbers on the two faces is even?

4) (10 points) Topic: **Probability spaces: darts**. We are throwing darts onto a disc of radius 1 and assume that the dart hits each region in the disc equally likely. What is the probability that two darts hit both a point in distance $\leq 1/2$ from the center.

5) (10 points) Topic: **Probability spaces: roulette**. A weel of circumferecne 2π is made into a probability space by assigning the probability that a ball lands in an arc of length s is $s/2\pi$. The weel is divided into 37 zones numbered $0, \dots 36$. The zones with even numbers except are painted black, while the odd numbers are painted red, The zone with number 0 is painted green. What is the probability that in two games, both red and black occur?