Math464/564 O. Knill, **March 1997**

Test:

Time: Thuesday, March 11, 1997, 14:00-14:45

<u>**Material**</u>: This test will be discussed during the second part of class and eventually during part of the next class. This test is on working techniques and will not be part of your grade

1) **Old mistakes** The following questions appeared in the first Midterm of this course and are only slightly changed. Do not look up the answer if you want to profit from this question.

01)		The union of a countable number of sets in a σ-algebra is in the σ-algebra.
02)		The set of all subsets of a set is a σ-algebra.
03)		A discrete random variable takes only finitely many values.
04)		It is always true that $\mathrm{P}[A\|B] \leq \mathrm{P}[A]$ if $\mathrm{P}[B] > 0$.

Having one or more mistakes here indicates that you should learn more from old mistakes.

2) **In class** The following things were mentioned only aside during class.

a) How old is probability theory?
b) What is the Banach-Tarsky paradox?
c) Why is the number 10^{100} called a "gogool"?
d) How does probability theory enter in quantum mechanics?
e) How did Euler define the "gradus suavitatis" for a frequency ratio.
f) Where do random variables with devilish distribution function appear in nature?
g) In which sense can one say that the harmonic series $\sum_{n=1}^{\infty} n^{-1}$ converges?
h) What is the geometric interpretation of the correlation coefficient?
i) Who found Chebychev's inequality before Chebychev?

If you should feel weak in this group of questions, try to do more careful notes during class (even if you know the topic already) or ask more questions during class. Aquire a technique to write fast and review each class for at least a half an hour before the next class.

3) **Intuition** Intuition helps to grasp formal definitions much easier. Having the right "picture" of an object in the mind allows you to figure out the definition without remembering it. That's one reason why computers are not yet good mathematicians.

How do you think about the following objects intuitively?

a) Ω

c) P
d) A random variable X
e) $\mathrm{E}[X]$
f) $\sigma(X)$
g) The covariance of two random variables X and Y.
h) The correlation coefficient
i) Independence.
k) Uncorrelated.

Feeling uncomfortable with this question might mean that you can improve your understanding by more thinking about the material or discussing the topics.

4) **Routine**.

a) 20 people are in a new year party. At midnight, they wish each other a happy good year. How many times, does one hear the clink of glasses?
b) Let X be the random variable which takes the value n with probability 2^{-n}. Compute the expectation of X.
c) Let X and Y be independent random variables. Express $\mathrm{E}[(X+Y)^3]$ in terms of moments of X and Y.
d) Let X be a random variable which takes the value 1 with probability 1/4 and the value -2 with probability 3/4. Compute its expectation.

If you are slow with answering these questions, you need more routine for solving easier problems. A good series of books to improve routine are "Schaum's outlines", where many problems with solutions are provided. Usually however, routine comes automatically.

5) **Yahoo**. In this question, your look-up performance is addressed.

a) Write down Bayes formula.
b) Find the probability generating function of the geometric distribution.
c) What is the switch on-switch off formula of Sylvester?
d) List the properties of the distribution function F_X?
e) What is the expectation of $X+Y+Z$, where X is Bernoulli(n=5,p=0.3) distributed, Y has geometric distribution with parameter 2 and Z is Poisson distributed with parameter 5?

If you are slow here, you can better organize the course material for example by concentrating the material onto a few pages, putting things into an organized binder.

6) **Big picture**.

a) Describe in three sentences what is "probability theory".
b) Which branches of mathematics do you know which are used in probability theory?
c) Which branches of science do you know which rely on probability theory?
d) Explain in three sentences to a interested layman what you are learning in probability theory.

Looking for the "big picture" is not everything and does not replace a detailed understanding. However, having the subject in the right "folder" helps considerably learning it. So it is a good idea to think from time to time about the big picture.

7) **Solving techniques**. Of how many of the following points are you aware of.

01) [ ] Can I solve a specific case of the problem (for example, if a problem has to be solved for general n, can I reformulate or solve it for $n = 1, 2, 3$?

02) [ ] Can I solve a more general problem? (Often, a specific case is psychologically harder than the general case, which might occur in the theory already).

03) [ ] I can reformulate the problem in my own words. (It might be that the obstacle is that the problem is not properly understood.

04) [ ] I ask myself actively: did I see a similar problem before? (This process usually runs automatically passively. It is worth giving it some more priority for a second.

05) [ ] I aks myself actively: to which topic does the problem belong? (Knowing where the problem belongs to is extremely helpful.).

06) [ ] Did I make at least two attempts to solve the problem? (If the problem can not be solved immediately, a second look after doing something else or sleeping over it can help.)

07) [ ] Did I sit over the problem long enough? Sometimes, it needs a while (a half an hour maybe) until the right "flash" of idea comes. Sitting over a problem without beeing able to solve it is an extremely important exercice (this actually happens most of the time when doing research).

08) [ ] Do I need a hint? Can I find the answer in a book or my notes? Can I formulate the question (for asking in an e-mail or in an office hour or in class). Can I guess a hint?

09) [ ] Are my working conditions good for me? (Light, table, enough sleep, coffeine, distraction by TV or Newspaper or background noise, enough paper and scratch paper for trying out ideas. Finding ideal working conditons need experimentation because because it is individual. It is worth to experiment with it.

10) [ ] Did I start early to think about the problem set. Starting late produces stress and can make it difficult to find the solution (starting an hour before having to turn in the problem set is too late for most people. A better idea is to solve a problem set early but with a watch. The later can help to improve performance under test conditions.

11) [ ] Do I think from time to time about improving my learning and solving techniques? For example, having bad physical working conditions can be an obstacle to find a solution. Evenso trivial, most people actually are not aware that techniques exist or that not being able to solve a problem immediately is very natural.

Final word: If you have spotted a point (or several points) which you feel needs improvement, try to work on this during the next weeks as well as in other classes. More stuff and material about problem solving techniques can be found in the library. A good book for mathematicians is

> "Polya, George, 1887-1985. "How to solve it; a new aspect of mathematical method", Princeton University Press, 1971

If you are short of time: most people have actually a lot of free CPU time awailable. One can think about mathematics (and other things) for example while biking or walking, trying to sleep, eating lunch, doing dishes, waiting in a line etc. So, keep always one or two "batch jobs" ready for processing during such "dead times".