Math 2^8: The Theory of Error-Correcting Codes

Tables of F, Δ, m_0, etc.

Type	q	F	Δ	q_0	$1/\Delta(q_0)$	m_0	δ	δ_{GV}
I	$(Y/X)^2$	$(1+q)^4$	$q(1-q)^2$	$1/3$	$27/4 = 6.75$	4	$1/4$	$.110\ldots$
II	$(Y/X)^4$	$(1+14q+q^2)^3$	$q(1-q)^4$	$1/5$	$5^5/4^4 = 12.207\ldots$	154	$1/6$	$.110\ldots$
III	$(Y/X)^3$	$(1+8q)^3$	$q(1-q)^3$	$1/4$	$256/27 = 9.481\ldots$	70	$1/4$	$.159\ldots$
IV	$(Y/X)^2$	$(1+3q)^3$	$q(1-q)^2$	$1/3$	$27/4 = 6.75$	17	$1/3$	$.189\ldots$
Λ	$e^{2\pi iz}$	E_4^3	$q\prod_{n=1}^{\infty}(1-q^n)^{24}$	$.0373-$	$69.116\ldots$	c.1700	*	*

Here m_0 is the smallest m for which a negative coefficient makes an extremal code impossible; δ is the asymptotic d/n for an extremal code of this type; and δ_{GV} is the asymptotic δ guaranteed by the Gilbert-Varshamov bound. The sphere-packing upper bound is $2\delta_{\mathrm{GV}}$. For even unimodular lattices ("Type Λ"), the aysmptotic $N_{\min}/m$ for an extremal lattice of rank $24m$ is 2, the G-V lower bound is $12/\pi e = 1.405\ldots$, and the sphere-packing upper bound is 4 times that.

In the generalization to Type `<numeral>` codes whose length is not a multiple of $\deg\Delta$, the bounds are as follows:

- Type I: $m_0 = 5, 6, 7$ for codes of length $n \equiv 2, 4, 6 \bmod 8$.
- Type II: $m_0 = 159, 164$ for codes of length $n \equiv 8, 16 \bmod 24$.
- Type III: $m_0 = 75, 78$ for codes of length $n \equiv 4, 8 \bmod 12$.
- Type IV: $m_0 = 20, 22$ for codes of length $n \equiv 2, 4 \bmod 6$.

For Type I (= odd unimodular) lattices, we take $q = e^{\pi iz}$, $F = \theta_{\mathbf{Z}}^8$, and $\Delta = \delta_8 = \big(\eta(q)\eta(q^4)/\eta(q^2)\big)^8$, and find:

$n \bmod 8$	0	1	2	3	4	5	6	7
m_0	4	5	6	7	8	8	9	10