

Math 155: Designs and groups

Possible final project topics

8 April 2016

There are any number of good final-project topics to be gleaned from the many pages of the textbook which we did not cover; some of you may also come up with good ideas from other sources (other courses, summer programs, independent reading, ...). Here are some specific ideas¹, though the list is at best representative, not exhaustive.

Golay codes A brief introduction to the theory of error-correcting codes; the Steiner (12,6,5) and/or (24,8,5) designs as manifestations of remarkable self-dual codes over $\mathbf{F}_3, \mathbf{F}_2$ respectively.

Spherical designs These are certain well-distributed configurations of points on the unit sphere in \mathbf{R}^n . How specifically are these analogous to our combinatorial structures also called “designs”?

Ovals and Segre’s theorem Develop Segre’s result that in a finite algebraic projective plane of odd order every oval is a conic.

Ovoids (see Example 1.42 on page 15) Explain the connection between inversive planes and ovoids in $\mathbf{P}^3(\mathbf{F}_q)$, and specifically between the inversive plane $\mathbf{P}^1(\mathbf{F}_{q^2})$ and elliptic quadrics, through at least one part of Theorem 1.43 on page 16.

Beyond Moore graphs Generalize the elementary lower bound $(q+1)^2 + 1$ on the size of a girth-5 graph with all degrees $> q$ to arbitrary girths. For girth 6, graphs attaining that bound $2(q^2 + q + 1)$ correspond to finite projective planes. Extending our eigenspace analysis that proved the 2-3-7-57 theorem for Moore graphs, show that equality cannot hold for any girths > 6 other than 8 and 12.

More remarkable strongly regular graphs (e.g. Schläfli, Gewirtz, McLaughlin, Higman-Sims): choose one and describe it and its automorphism group in some detail.

Symplectic groups and/or 8-cages Develop the theory of symplectic spaces: vector spaces of finite (and necessarily even) dimension with an *alternating* bilinear pairing (so $\langle x, x \rangle = 0$ for all vectors x , whence also $\langle x, y \rangle = \langle y, x \rangle$ for all x, y). The automorphism group of such a space is called the symplectic group $\mathrm{Sp}_{2n}(F)$, which is $\mathrm{SL}_2(F)$ for $n = 1$ but new for $n \geq 2$. When F is finite, compute the group order, and show that $\mathrm{PSP}_{2n}(F)$ is simple except for the two known exceptions with $n = 1$ and the coincidence $\mathrm{Sp}_4(\mathbf{F}_2) \cong S_6$. Alternatively, use symplectic spaces of dimension 4 to construct minimal graphs of degree $q + 1$ and girth 8 (the Tutte 8-cage is the case $F = \mathbf{F}_2$ of this construction).

Whatever topic you choose, be sure to clear it with me beforehand and to meet with me at least once before writing your final paper. *This final project is not collaborative* — you may ask friends to read drafts (but if you’re doing that, you might as well ask me or the CA), but must both learn the math and write it up on your own. But it’s not an in-class midterm exam: you may and should consult and learn from references (and of course acknowledge them properly and use your own words in the paper).

¹Which could also be starting points for honors theses.