

Math 155: Designs and groups

Possible final project topics

12 April 2010

There are any number of good final-project topics to be gleaned from the many pages of the textbook which we did not cover; some of you may also come up with good ideas from other sources (other courses, summer programs, independent reading, . . .). Here are some specific ideas¹, though the list is at best representative, not exhaustive.

Golay codes A brief introduction to the theory of error-correcting codes; the Steiner (12,6,5) and/or (24,8,5) designs as manifestations of remarkable self-dual codes over $\mathbf{F}_3, \mathbf{F}_2$ respectively.

Spherical designs These are certain well-distributed configurations of points on the unit sphere in \mathbf{R}^n . How specifically are these analogous to our combinatorial structures also called “designs”?

Ovals and Segre’s theorem Develop Segre’s result that in a finite algebraic projective plane of odd order every oval is a conic.

Ovoids (see Example 1.42 on page 15) Explain the connection between inversive planes and ovoids in $\mathbf{P}^3(\mathbf{F}_q)$, and specifically between the inversive plane $\mathbf{P}^1(\mathbf{F}_{q^2})$ and elliptic quadrics, through at least one part of Theorem 1.43 on page 16.

Exotic hyperovals and ovoids Once $\alpha \geq 4$, the projective plane over the field of 2^α elements contains hyperovals not of the form conic+center. We’ve seen a nice family of such hyperovals for $\alpha \neq 1, 2, 3, 4, 6$. Describe such a hyperoval for $\alpha = 4$ and/or $\alpha = 6$. Alternatively, describe the Suzuki-Tits ovoids in $\mathbf{P}^3(\mathbf{F}_q)$ when q is an odd power of 2 (which are connected with Suzuki’s simple subgroup of $\text{Sp}_4(\mathbf{F}_q)$).

More remarkable strongly regular graphs (e.g. Schläfli, Gewirtz, McLaughlin, Higman-Sims): choose one and describe it and its automorphism group in some detail.

Whatever topic you choose, be sure to clear it with me beforehand and to meet with me at least once before writing your final paper.

¹Which could also be starting points for honors theses.