Riemann surfaces, dynamics and geometry

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7 Problems: Hyperbolic manifolds

1. Let M be a closed hyperbolic manifold. Prove the geodesic flow is ergodic on the frame bundle of M.

2. Let N^3 be a convex hyperbolic 3-manifold. Prove ∂N is incompressible iff the limit set of N is connected.

3. Suppose ∂N is compressible. Give a direct proof of Dehn's lemma: that is, show there is a properly embedded disk $(D^2, \partial D^2) \subset (M, \partial M)$ with $\pi_1(\partial D^2)$ nontrivial in $\pi_1(\partial M)$.