Math 264 — Homework #5

Due Monday, March 4, 2002

1. We've asserted in class that the Chow ring/cohomology ring of the Grassmannian $\mathbb{G}(1,3)$ is generated by the Chern classes of the universal quotient bundle, with relations obtained by setting the third and higher Chern classes of the universal subbundle equal to zero: that is,

$$A^*(\mathbb{G}) \;=\; \mathbb{Z}[\sigma_1, \sigma_2] / \left([\frac{1}{1+\sigma_1+\sigma_2}]_\alpha = 0 \quad \forall \alpha > 2 \right).$$

Verify that this agrees with the description of $A^*(\mathbb{G})$ we gave earlier.

2. Let $S \subset \mathbb{P}^3$ be a general surface of degree d. How many lines in $\mathbb{P}^3$ have a point of contact of order 5 or more with S?

Remember that to fully solve this problem you need to both calculate the number, and also to verify that for general S there are indeed only finitely many such lines (this is relatively easy) and that each such line counts with multiplicity one (slightly harder). Of course, you don't need to do the problem at all, so I guess if you want to just have the fun of computing the number and leave the tedious verifications alone, that's OK too.

3. Let $\{C_\lambda\}_{\lambda \in \mathbb{P}^1}$ be a general pencil of plane quartic curves. How many curves in the pencil have hyperflexes—that is, points at which the tangent line has contact of order 4 or more with the curve?

For a real challenge, try this: find the degree and genus of the curve traced out by the flexes of the curves C_λ.