Math 264 — Homework #10

Due Monday, May 6, 2002

These are somewhat harder exercises than the typical homework problems we've done this semester, so just do two of them.

1. Let $\pi : X \to Y$ be a double cover (that is, a finite flat morphism of degree 2) of varieties.
   a. Describe the direct image $\pi_* \mathcal{O}_X$ of the structure sheaf of X.
   b. Use this to classify double covers of a given variety Y.

2. The standard proof of the Grothendieck Riemann Roch formula is in two stages: you prove it for an embedding $X \hookrightarrow B$, and for a projective bundle $X \to B$; since every projective morphism factors as a composition of these, this suffices. In this exercise, we're going to do the second case.
   a. Let $\pi : X = \mathbb{P}E \to B$ be a projective bundle over a smooth variety B. Show that the GRR for an arbitrary coherent sheaf $\mathcal{F}$ on X follows from the GRR for powers S^k of the universal subbundle S on X.
   b. Now prove the formula in this case.

3. As we've said in class, the definition of Chern classes may be extended from vector bundles to arbitrary coherent sheaves on a smooth variety X (any coherent sheaf has a resolution by locally free sheaves). Let X be a smooth n-dimensional variety, $p \in X$ a point and $\mathcal{F} = \mathcal{O}_{\{p\}}$ the structure sheaf of $\{p\}$, viewed as a coherent sheaf on X. Find the Chern classes of $\mathcal{F}$. (Hint: use Grothendieck Riemann Roch!)

4. Without using Porteous' formula, prove that any compact Riemann surface of genus 6 has a nonconstant rational function of degree 4 or less.