

CHAPTER ONE

Solutions for Section 1.1

Exercises

- $f(6.9) = 2.9$.
- $(2.2, 2.9); (6.1, 4.9)$
- Since $f(0) = f(4) = f(8) = 0$, the solutions are $x = 0, 4, 8$.
- Since the graphs touch at $x = 2.2$ and $x = 6.1$, these are the solutions.
- $m = f(v)$.
- $w = f(c)$.
- (a) The graphs in (I), (III), (IV), (V), (VII), and (VIII) are functions. The graphs in (II), (VI), and (IX) do not pass the vertical line test and so they cannot be the graphs of functions.
 (b) (i) The graph of SAT Math score versus SAT Verbal score for a number of students will be a graph of a number of points. Graphs (V) and (VI) are of this type.
 (ii) The graph of hours of daylight per day must be an oscillating function (since the number of hours of daylight fluctuates up and down throughout the year). Graph (VIII) represents this.
 (c) If the train fare remains constant throughout the day, graph (III) describes the fare. If there are specific times of the day (rush hours, for example) when the train company raises its prices, then graph (IV) represents the train fare as a function of time of day.
- These data are plotted in Figure 1.1. The independent variable is A and the dependent variable is n .

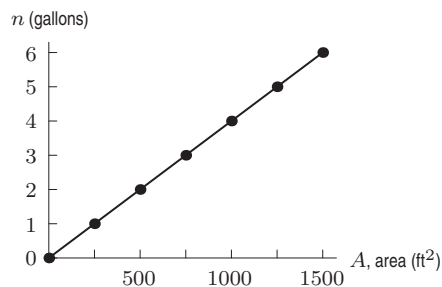


Figure 1.1

- (a) Since the vertical intercept is $(0, 40)$, we have $f(0) = 40$.
 (b) Since the horizontal intercept is $(2, 0)$, we have $f(2) = 0$.
- (a) Since $f(x)$ is 4 when $x = 0$, we have $f(0) = 4$.
 (b) Since $x = 3$ when $f(x) = 0$, we have $f(3) = 0$.
 (c) $f(1) = 2$
 (d) There are two x values leading to $f(x) = 1$, namely $x = 2$ and $x = 4$. So $f(2) = 1$ and $f(4) = 1$.
- (a) w goes on the horizontal axis
 (b) $(-4, 10)$
 (c) $(6, 1)$

12. (a) We have

$$f(0) = \frac{10}{1+0^2} = \frac{10}{1} = 10$$

$$f(1) = \frac{10}{1+1^2} = \frac{10}{2} = 5$$

$$f(2) = \frac{10}{1+2^2} = \frac{10}{5} = 2$$

$$f(3) = \frac{10}{1+3^2} = \frac{10}{10} = 1.$$

See Table 1.1.

Table 1.1

x	0	1	2	3
$f(x)$	10	5	2	1

- (b) For
- $x = 0$
- , we have
- $f(0) = 10$
- . This value is largest because the
- x
- value is smallest.

13. Appropriate axes are shown in Figure 1.2.

14. Appropriate axes are shown in Figure 1.3.

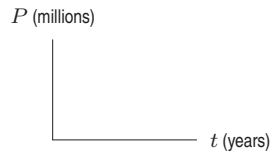


Figure 1.2

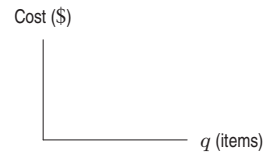


Figure 1.3

15. Appropriate axes are shown in Figure 1.4.

16. Appropriate axes are shown in Figure 1.5.

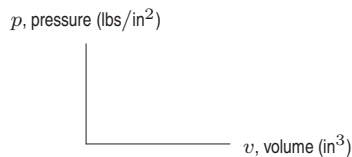


Figure 1.4

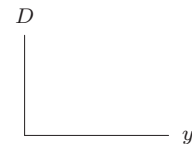


Figure 1.5

Problems

17. (a) When there is no snow, it is equivalent to no rain. Thus, the vertical intercept is 0. Since every ten inches of snow is equivalent to one inch of rain, we can specify the slope as

$$\frac{\Delta \text{rain}}{\Delta \text{snow}} = \frac{1}{10} = 0.1$$

We have a vertical intercept of 0 and a slope of 0.1. Thus, the equation is: $r = f(s) = 0.1s$.

- (b) By substituting 5 in for s , we get $f(5) = 0.1(5) = 0.5$. This tells us that five inches of snow is equivalent to approximately 1/2 inch of rain.
- (c) Substitute 5 inches for $r = f(s)$ in the equation: $5 = 0.1s$. Solving gives $s = 50$. Five inches of rain is equivalent to approximately 50 inches of snow.

18. (a) One, because otherwise it would automatically fail the vertical-line test using the y -axis as the vertical line.
 (b) Yes, it can cross an infinite number of times. For example, the graph in Figure 1.6 oscillates an infinite number of times across the x -axis.

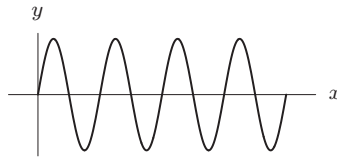
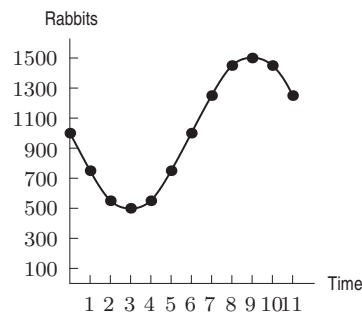
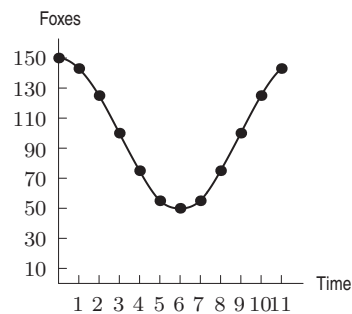


Figure 1.6

19. (a) The number of people who own cell phones in the year 2000 is 100,300,000.
 (b) There are 20,000,000 people who own cell phones a years after 1990.
 (c) There will be b million people who own cell phones in the year 2010.
 (d) The number n is the number of people (in millions) who own cell phones t years after 1990.
20. From the table, $r(300) = 120$, which tells us that at a height of 300 m the wind speed is 120 mph.
21. Judging from the table, $r(s) \geq 116$ for $200 \leq s \leq 1000$. This tells us that the wind speed is at least 116 mph between 200 m and 1000 m above the ground.
22. The wind reaches its greatest speed, $v = 122$ mph, at a height of $s = 500$ m.
23. (a) 69°F
 (b) July 17th and 20th
 (c) Yes. For each date, there is exactly one low temperature.
 (d) No, it is not true that for each low temperature, there is exactly one date: for example, 73° corresponds to both the 17th and 20th.
24. (a) From the table, we see that $f(100) = 524.5$. This means that there is approximately \$524.5 billion worth of \$100 bills in circulation in the United States.
 (b) To determine the number of \$5 bills, we divide 9.7 by 5. Thus, we have about 1.94 billion \$5 bills in circulation. The number of \$1 bills is the same as the value, so there are 8.4 billion \$1 bills in circulation. There are more \$1 bills.
25. (a) Figure 1.7 shows the plot of R versus t . R is a function of t because no vertical line intersects the graph in more than one place.
 (b) Figure 1.8 shows the plot of F versus t . F is a function of t because no vertical line intersects the graph in more than one place.

Figure 1.7: The graph of R versus t Figure 1.8: The graph of F versus t

- (c) Figure 1.9 shows the plot of F versus R . We have also drawn the vertical line corresponding to $R = 567$. This tells us that F is not a function of R because there is a vertical line that intersects the graph twice. In fact the lines $R = 567$, $R = 750$, $R = 1000$, $R = 1250$, and $R = 1433$ all intersect the graph twice. However, the existence of any one of them is enough to guarantee that F is not a function of R .

- (d) Figure 1.10 shows the plot of R versus F . We have drawn the vertical line corresponding to $F = 57$. This tells us that R is not a function of F because there is a vertical line that intersects the graph twice. In fact the lines $F = 57$, $F = 75$, $F = 100$, $F = 125$, and $F = 143$ all intersect the graph twice. However, the existence of any one of them is enough to guarantee that R is not a function of F .

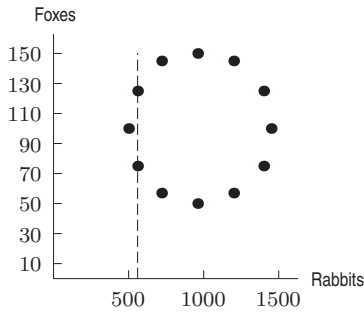


Figure 1.9: The graph of F versus R

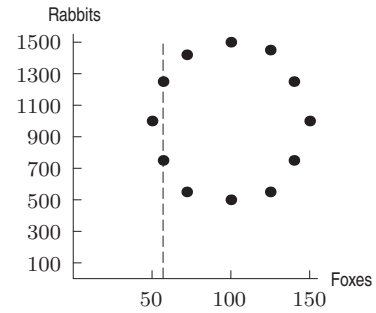


Figure 1.10: The graph of R versus F

26. (a) No, in the year 1954 there were two world records; in the year 1981 there were three world records.
 (b) Yes, each world record occurred in only one year.
 (c) The world record of 3 minutes and 47.33 seconds was set in 1981.
 (d) The statement $y(3:51.1) = 1967$ tells us that the world record of 3 minutes, 51.1 seconds was set in 1967.
27. (a) Yes, if we know the Congress we can determine the number of female senators.
 (b) No, there were 2 female senators in the 98th, 100th and 102nd Congresses.
 (c) The number of female senators who served the 104th Congress is 8.
 (d) The statement $S(108) = 14$ tells us that 14 female senators served the 108th Congress.
28. A possible graph is shown in Figure 1.11.
29. A possible graph is shown in Figure 1.12.

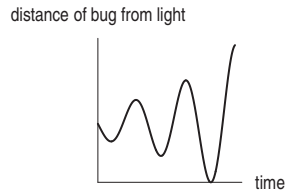


Figure 1.11

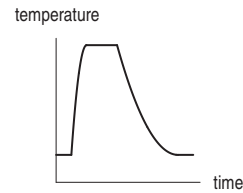


Figure 1.12

30. Since the tax is $0.06P$, the total cost would be the price of the item plus the tax, or

$$C = P + 0.06P = 1.06P.$$

31. The area of each end of the can is πr^2 . To find the surface area of the cylindrical side, imagine making vertical cut from top to bottom and unfolding the cylinder into a rectangle. See Figure 1.13.

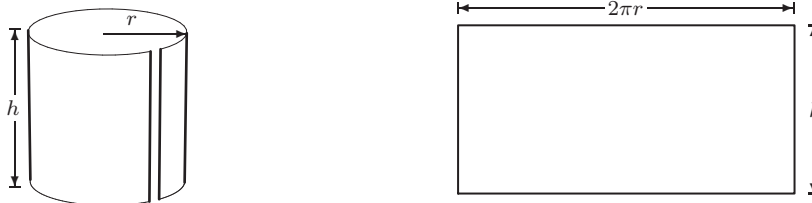


Figure 1.13

Thus, the surface area of the cylindrical side is $2\pi rh$.
The total surface area of the can is given by

$$S = 2(\text{Area of one end}) + \text{Area of cylindrical side}$$

$$S = 2(\pi r^2) + 2\pi rh.$$

Using the fact that height is twice radius, $h = 2r$, we get

$$S = 2\pi r^2 + 2\pi r(2r) = 6\pi r^2.$$

32. (a) It takes Charles Osgood 60 seconds to read 15 lines, so that means it takes him 4 seconds to read 1 line, 8 seconds for 2 lines, and so on. Table 1.2 shows this. From the table we see that it takes 36 seconds to read 9 lines.

Table 1.2 *The time it takes Charles Osgood to read*

Lines	0	1	2	3	4	5	6	7	8	9	10
Time	0	4	8	12	16	20	24	28	32	36	40

- (b) Figure 1.14 shows the plot of the time in seconds versus the number of lines.
(c) In Figure 1.15 we have dashed in a line to see the trend. By drawing the vertical line at 9 lines, we see that this corresponds to approximately 36 seconds. By drawing a horizontal line at 30 seconds, we see that this corresponds to approximately 7.5 lines.

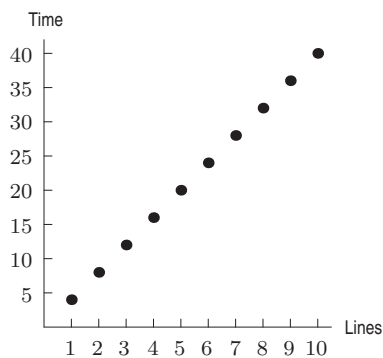


Figure 1.14: The graph of time versus lines

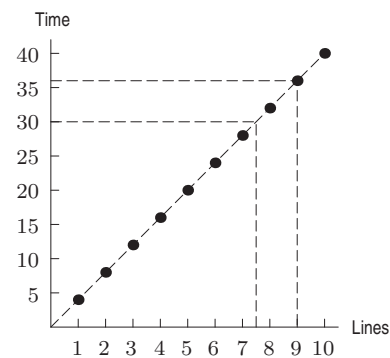


Figure 1.15: The graph of time versus lines

- (d) If we let T be the time in seconds that it takes to read n lines, then $T = 4n$.
33. (a) Since the person starts out 5 miles from home, the vertical intercept on the graph must be 5. Thus, (i) and (ii) are possibilities. However, since the person rides 5 mph away from home, after 1 hour the person is 10 miles from home. Thus, (ii) is the correct graph.
- (b) Since this person also starts out 5 miles from home, (i) and (ii) are again possibilities. This time, however, the person is moving at 10 mph and so is 15 miles from home after 1 hour. Thus, (i) is correct.
- (c) The person starts out 10 miles from home so the vertical intercept must be 10. The fact that the person reaches home after 1 hour means that the horizontal intercept is 1. Thus, (v) is correct.
- (d) Starting out 10 miles from home means that the vertical intercept is 10. Being half way home after 1 hour means that the distance from home is 5 miles after 1 hour. Thus, (iv) is correct.
- (e) We are looking for a graph with vertical intercept of 5 and where the distance is 10 after 1 hour. This is graph (ii). Notice that graph (iii), which depicts a bicyclist stopped 10 miles from home, does not match any of the stories.

34. (a)

Table 1.3 Relationship between cost, C , and number of liters produced, l

l (millions of liters)	0	1	2	3	4	5
C (millions of dollars)	2.0	2.5	3.0	3.5	4.0	4.5

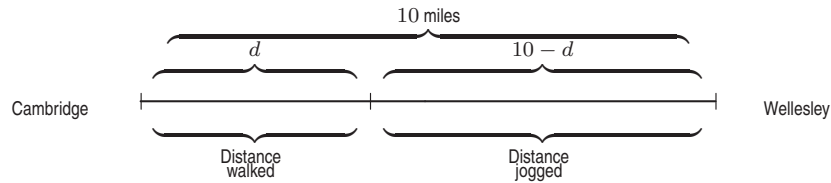
(b) The cost, C , consists of a fixed cost of \$2 million plus a variable cost of \$0.50 million per million liters produced. If l millions of liters are produced, the total variable costs are $(0.5)l$. Thus, the total cost C in millions of dollars is given by

$$C = \text{Fixed cost} + \text{Variable cost},$$

so

$$C = 2 + (0.5)l.$$

35. The diagram is shown in Figure 1.16.

**Figure 1.16**

The total time the trip takes is given by the equation

$$\text{Total time} = \text{Time walked} + \text{Time jogged}.$$

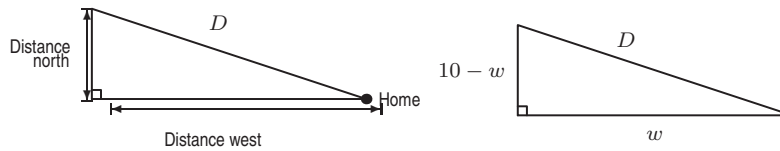
The distance walked is d , and, since the total distance is 10, the remaining distance jogged is $(10 - d)$. See Figure 1.16. We know that time equals distance over speed, which means that

$$\text{Time walked} = \frac{d}{5} \quad \text{and} \quad \text{Time jogged} = \frac{10 - d}{8}.$$

Thus, the total time is given by the equation

$$T(d) = \frac{d}{5} + \frac{10 - d}{8}.$$

36. (a) Yes. If the person walks due west and then due north, the distance from home is represented by the hypotenuse of the right triangle that is formed (see Figure 1.17).

**Figure 1.17**

If the distance west is w miles and the total distance walked is 10 miles, then the distance north is $10 - w$ miles. We can use the Pythagorean Theorem to find that

$$D = \sqrt{w^2 + (10 - w)^2}.$$

So, for each value of w , there is a unique value of D given by this formula. Thus, the definition of a function is satisfied.

- (b) No. Suppose she walks 10 miles, that is, $x = 10$. She might walk 1 mile west and 9 miles north, or 2 miles west and 8 miles north, or 3 miles west and 7 miles north, and so on. The right triangles in Fig 1.18 show three different routes she could take and still walk 10 miles.

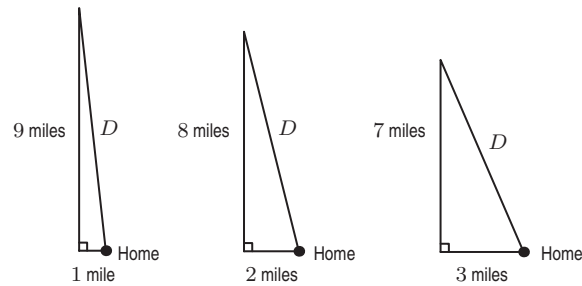


Figure 1.18

Each situation gives a different distance from home. The Pythagorean Theorem shows that the distances from home for these three examples are

$$D = \sqrt{1^2 + 9^2} = 9.06,$$

$$D = \sqrt{2^2 + 8^2} = 8.25,$$

$$D = \sqrt{3^2 + 7^2} = 7.62.$$

Thus, the distance from home cannot be determined from the distance walked.

Solutions for Section 1.2

Exercises

1. We have $G(3) - G(-1) > 0$.
2. We have $F(-2) > F(2)$.
3. We have

$$\begin{aligned} \frac{\Delta f}{\Delta x} &= \frac{f(6.1) - f(2.2)}{6.1 - 2.2} \\ &= \frac{4.9 - 2.9}{3.9} \\ &= 0.513. \end{aligned}$$

4. There are many such intervals. One way to find them is to look for points at which a horizontal line (a line of slope $m = 0$) intersects the graph. Here are some possible intervals: $0 \leq x \leq 4$, $0 \leq x \leq 8$, $4 \leq x \leq 8$, $2.2 \leq x \leq 5.2$, $2.2 \leq x \leq 6.9$, $5.2 \leq x \leq 6.9$.
5. Using the points on g

$$\text{Average rate of change} = \frac{g(6.1) - g(2.2)}{6.1 - 2.2} = \frac{4.9 - 2.9}{6.1 - 2.2} = 0.513.$$

6. They are equal; both are given by

$$\frac{4.9 - 2.9}{6.1 - 2.2}$$

7. (a) Negative
-
- (b) Positive

8. (a) Let
- $s = V(t)$
- be the sales (in millions) of VCRs in year
- t
- . Then

$$\begin{aligned} \text{Average rate of change of } s \text{ from } t = 1998 \text{ to } t = 2000 &= \frac{\Delta s}{\Delta t} = \frac{V(2000) - V(1998)}{2000 - 1998} \\ &= \frac{1869 - 2409}{2} \\ &= -270 \text{ million VCRs/year.} \end{aligned}$$

Let $q = D(t)$ be the sales (in millions) of DVD players in year t . Then

$$\begin{aligned} \text{Average rate of change of } q \text{ from } t = 1998 \text{ to } t = 2000 &= \frac{\Delta q}{\Delta t} = \frac{D(2000) - D(1998)}{2000 - 1998} \\ &= \frac{1717 - 421}{2} \\ &= 648 \text{ million DVD players/year.} \end{aligned}$$

- (b) By the same argument

$$\begin{aligned} \text{Average rate of change of } s \text{ from } t = 2000 \text{ to } t = 2003 &= \frac{\Delta s}{\Delta t} = \frac{V(2003) - V(2000)}{2003 - 2000} \\ &= \frac{407 - 1869}{3} \\ &= -487.33 \text{ million VCRs/year.} \end{aligned}$$

$$\begin{aligned} \text{Average rate of change of } q \text{ from } t = 2000 \text{ to } t = 2003 &= \frac{\Delta q}{\Delta t} = \frac{D(2003) - D(2000)}{2003 - 2000} \\ &= \frac{3050 - 1717}{3} \\ &= 444.33 \text{ million DVD players/year.} \end{aligned}$$

- (c) The fact that $\Delta s/\Delta t = -270$ tells us that VCR sales decreased at an average rate of 270 million VCRs/year between 1998 and 2000. The fact that the average rate of change is negative tells us that annual sales are decreasing. The fact that $\Delta s/\Delta t = -487.33$ tells us that VCR sales decreased at an average rate of 487.33 million VCRs/year between 2000 and 2003. The fact that $\Delta q/\Delta t = 648$ means that DVD player sales increased at an average rate of 648 million players/year between 1998 and 2000. The fact that $\Delta q/\Delta t = 444.33$ means that LP sales increased at an average rate of 444.33 million players/year between 2000 and 2003.
9. To decide if VCR sales are an increasing or decreasing function of DVD player sales, we must read the table in the direction in which DVD player sales increase. This means we read the table from right to left. As the number of DVD player sales increases, the number of VCR sales decrease. Thus, VCR sales are a decreasing function of DVD player sales.

10. (a) (i) After 2 hours 60 miles had been traveled. After 5 hours, 150 miles had been traveled. Thus on the interval from
- $t = 2$
- to
- $t = 5$
- the value of
- Δt
- is

$$\Delta t = 5 - 2 = 3$$

and the value of ΔD is

$$\Delta D = 150 - 60 = 90.$$

- (ii) After 0.5 hours 15 miles had been traveled. After 2.5 hours, 75 miles had been traveled. Thus on the interval from $t = 0.5$ to $t = 2.5$ the value of Δt is

$$\Delta t = 2.5 - .5 = 2$$

and the value of ΔD is

$$\Delta D = 75 - 15 = 60.$$

- (iii) After 1.5 hours 45 miles had been traveled. After 3 hours, 90 miles had been traveled. Thus on the interval from $t = 1.5$ to $t = 3$ the value of Δt is

$$\Delta t = 3 - 1.5 = 1.5$$

and the value of ΔD is

$$\Delta D = 90 - 45 = 45.$$

- (b) For the interval from $t = 2$ to $t = 5$, we see

$$\text{Rate of change} = \frac{\Delta D}{\Delta t} = \frac{90}{3} = 30.$$

For the interval from $t = 0.5$ to $t = 2.5$, we see

$$\text{Rate of change} = \frac{\Delta D}{\Delta t} = \frac{60}{2} = 30.$$

For the interval from $t = 1.5$ to $t = 3$, we see

$$\text{Rate of change} = \frac{\Delta D}{\Delta t} = \frac{45}{1.5} = 30.$$

This suggests that the average speed is 30 miles per hour throughout the trip.

11. (a) For 1990 to 2000, the rate of change of P_1 is

$$\frac{\Delta P_1}{\Delta t} = \frac{83 - 53}{2000 - 1990} = \frac{30}{10} = 3 \text{ hundred people per year,}$$

while for P_2 we have

$$\frac{\Delta P_2}{\Delta t} = \frac{70 - 85}{2000 - 1990} = \frac{-15}{10} = -1.5 \text{ hundred people per year.}$$

- (b) For 1995 to 2007,

$$\frac{\Delta P_1}{\Delta t} = \frac{93 - 73}{2007 - 1995} = \frac{20}{12} = 1.67 \text{ hundred people per year,}$$

and

$$\frac{\Delta P_2}{\Delta t} = \frac{65 - 75}{2007 - 1995} = \frac{-10}{12} = -0.83 \text{ hundred people per year.}$$

- (c) For 1990 to 2007,

$$\frac{\Delta P_1}{\Delta t} = \frac{93 - 53}{2007 - 1990} = 2.35 \text{ hundred people per year,}$$

and

$$\frac{\Delta P_2}{\Delta t} = \frac{65 - 85}{2007 - 1990} = -1.18 \text{ hundred people per year.}$$

12. (a) (i) We find the average rate of change in the population as follows. For P_1 from 1990 to 2000,

$$\text{Rate of change} = \frac{\Delta P_1}{\Delta t} = \frac{62 - 42}{2000 - 1990} = 2 \text{ thousand people per year.}$$

Thus, P_1 is growing, on average, by two thousand people per year. For P_2 over the same period,

$$\text{Rate of change} = \frac{\Delta P_2}{\Delta t} = \frac{72 - 82}{2000 - 1990} = -1 \text{ thousand people per year.}$$

The negative sign tells us that P_2 is decreasing, on average, by one thousand people per year.

(ii) For 1990–2007, the average rate of change of P_1 is:

$$\text{Rate of change} = \frac{\Delta P_1}{\Delta t} = \frac{76 - 42}{2007 - 1990} = 2 \text{ thousand people per year.}$$

That is, the city is gaining 2 thousand people per year. The average rate of change of P_2 is:

$$\text{Rate of change} = \frac{\Delta P_2}{\Delta t} = \frac{65 - 82}{2007 - 1990} = -1 \text{ thousand people per year.}$$

That is, the city is losing a thousand people per year.

(iii) For 1995–2007, we have:

$$\frac{\Delta P_1}{\Delta t} = \frac{76 - 52}{2007 - 1995} = 2 \text{ thousand people per year.}$$

That is, the city is gaining 2 thousand people per year. The average rate of growth for the second population is:

$$\frac{\Delta P_2}{\Delta t} = \frac{65 - 77}{2007 - 1995} = -1 \text{ thousand people per year.}$$

That is, the city is losing a thousand people per year.

- (b) The average rate of change of each population is the same on all three time intervals. Each population appears to be changing at a constant rate. The first population is growing, on average, by 2 thousand people per year in each time interval. The second population is dropping, on average, by 1 thousand people per year in each time interval.

Problems

13. According to the table in the text, the tree has $139\mu\text{g}$ of carbon-14 after 3000 years from death and $123\mu\text{g}$ of carbon-14 after 4000 years from death. Because the function $L = g(t)$ is decreasing, the tree must have died between 3,000 and 4,000 years ago.
14. (a) According to the table, a 200-lb person uses 5.4 calories per minute while walking. Since a half hour is 30 minutes, a half-hour walk uses $(5.4)(30) = 162$ calories.
- (b) A 120-lb swimmer uses 6.9 calories per minute. Thus, in one hour the swimmer uses $(6.9)(60) = 414$ calories. A 220-lb bicyclist uses 11.9 calories per minute. In a half-hour, the bicyclist uses $(11.9)(30) = 357$ calories. Thus, the swimmer uses more calories.
- (c) Increases, since the numbers 2.7, 3.2, 4.0, 4.6, 5.4, 5.9 are increasing.
15. (a) Between $(-2, -7)$ and $(3, 3)$,

$$\text{Average rate of change} = \frac{\Delta y}{\Delta x} = \frac{3 - (-7)}{3 - (-2)} = \frac{10}{5} = 2.$$

- (b) The function is increasing over this interval, since the average rate of change is positive.
- (c) As the x -values increase, so do the y -values. See Figure 1.19. Thus, this function is increasing everywhere.

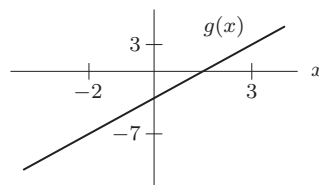


Figure 1.19

16. (a) (i) We have

$$\frac{f(2) - f(0)}{2 - 0} = \frac{16 - 2^2 - (16 - 0)}{2} = -\frac{4}{2} = -2.$$

This means $f(x)$ decreases by an average of 2 units per unit change in x on the interval $0 \leq x \leq 2$.

- (ii) We have

$$\frac{f(4) - f(2)}{4 - 2} = \frac{16 - (4)^2 - (16 - 2^2)}{2} = \frac{-16 + 4}{2} = -6.$$

This means $f(x)$ decreases by an average of 6 units per unit change in x on the interval $2 \leq x \leq 4$.

- (iii) We have

$$\frac{f(4) - f(0)}{4 - 0} = \frac{16 - (4)^2 - (16 - 0)}{4} = -\frac{16}{4} = -4.$$

This means $f(x)$ decreases by an average of 4 units per unit change in x on the interval $0 \leq x \leq 4$.

- (b) The graph of $f(x)$ is the solid curve in Figure 1.20. The secants corresponding to each rate of change are shown as dashed lines. The average rate of decrease is greatest on the interval $2 \leq x \leq 4$.

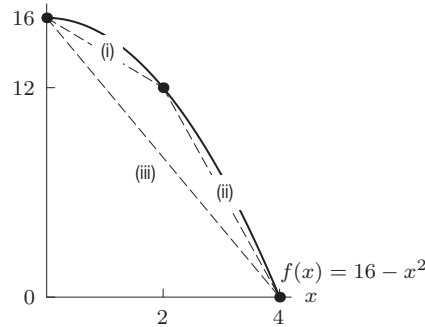


Figure 1.20

17. (a) From the graph, we see that $g(4) \approx 2$ and $g(0) \approx 0$. Thus,

$$\frac{g(4) - g(0)}{4 - 0} \approx \frac{2 - 0}{4 - 0} = \frac{1}{2}.$$

- (b) The line segment joining the points in part (a), as well as the line segment in part (d), is shown on the graph in Figure 1.21.

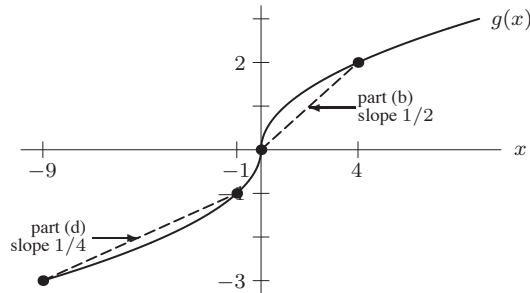


Figure 1.21

- (c) From the graph, $g(-9) \approx -3$ and $g(-1) \approx -1$. Thus,

$$\frac{g(b) - g(a)}{b - a} \approx \frac{-1 - (-3)}{-1 - (-9)} = \frac{2}{8} = \frac{1}{4}.$$

- (d) The line segment in part (c) with slope $(1/4)$ is shown in Figure 1.21.

18. (a) (i) Between
- $(-1, f(-1))$
- and
- $(3, f(3))$

$$\text{Average rate of change} = \frac{f(3) - f(-1)}{3 - (-1)} = \frac{(15 - 4) - (-5 - 4)}{4} = \frac{11 - (-9)}{4} = \frac{20}{4} = 5.$$

- (ii) Between
- $(a, f(a))$
- and
- $(b, f(b))$

$$\text{Average rate of change} = \frac{f(b) - f(a)}{b - a} = \frac{(5b - 4) - (5a - 4)}{b - a} = \frac{5b - 4 - 5a + 4}{b - a} = \frac{5b - 5a}{b - a} = \frac{5(b - a)}{b - a} = 5.$$

- (iii) Between
- $(x, f(x))$
- and
- $(x + h, f(x + h))$

$$\begin{aligned} \text{Average rate of change} &= \frac{f(x + h) - f(x)}{(x + h) - x} = \frac{(5(x + h) - 4) - (5x - 4)}{(x + h) - x} \\ &= \frac{5x + 5h - 4 - 5x + 4}{h} = \frac{5h}{h} = 5. \end{aligned}$$

- (b) The average rate of change is always 5.

19. (a) (i) Between
- $(-1, f(-1))$
- and
- $(3, f(3))$

$$\text{Average rate of change} = \frac{f(3) - f(-1)}{3 - (-1)} = \frac{\left(\frac{3}{2} + \frac{5}{2}\right) - \left(\frac{-1}{2} + \frac{5}{2}\right)}{4} = \frac{4 - 2}{4} = \frac{2}{4} = \frac{1}{2}.$$

- (ii) Between
- $(a, f(a))$
- and
- $(b, f(b))$

$$\text{Average rate of change} = \frac{f(b) - f(a)}{b - a} = \frac{\left(\frac{b}{2} + \frac{5}{2}\right) - \left(\frac{a}{2} + \frac{5}{2}\right)}{b - a} = \frac{\frac{b}{2} + \frac{5}{2} - \frac{a}{2} - \frac{5}{2}}{b - a} = \frac{\frac{b - a}{2}}{b - a} = \frac{\frac{1}{2}(b - a)}{b - a} = \frac{1}{2}.$$

- (iii) Between
- $(x, f(x))$
- and
- $(x + h, f(x + h))$

$$\begin{aligned} \text{Average rate of change} &= \frac{f(x + h) - f(x)}{(x + h) - x} = \frac{\left(\frac{x+h}{2} + \frac{5}{2}\right) - \left(\frac{x}{2} + \frac{5}{2}\right)}{(x + h) - x} \\ &= \frac{\frac{x+h}{2} + \frac{5}{2} - \frac{x}{2} - \frac{5}{2}}{x + h - x} = \frac{\frac{x+h-x}{2}}{h} = \frac{\frac{h}{2}}{h} = \frac{1}{2}. \end{aligned}$$

- (b) The average rate of change is always
- $\frac{1}{2}$
- .

20. (a) (i) Between
- $(-1, f(-1))$
- and
- $(3, f(3))$

$$\text{Average rate of change} = \frac{f(3) - f(-1)}{3 - (-1)} = \frac{(3^2 + 1) - ((-1)^2 + 1)}{4} = \frac{10 - 2}{4} = \frac{8}{4} = 2.$$

- (ii) Between
- $(a, f(a))$
- and
- $(b, f(b))$

$$\begin{aligned} \text{Average rate of change} &= \frac{f(b) - f(a)}{b - a} = \frac{(b^2 + 1) - (a^2 + 1)}{b - a} \\ &= \frac{b^2 + 1 - a^2 - 1}{b - a} = \frac{b^2 - a^2}{b - a} = \frac{(b + a)(b - a)}{b - a} = b + a. \end{aligned}$$

- (iii) Between
- $(x, f(x))$
- and
- $(x + h, f(x + h))$

$$\begin{aligned} \text{Average rate of change} &= \frac{f(x + h) - f(x)}{(x + h) - x} = \frac{((x + h)^2 + 1) - (x^2 + 1)}{(x + h) - x} \\ &= \frac{x^2 + 2xh + h^2 + 1 - x^2 - 1}{x + h - x} = \frac{2xh + h^2}{h} = \frac{h(2x + h)}{h} = 2x + h. \end{aligned}$$

- (b) The average rate of change is different each time. However, it seems to be the sum of the two
- x
- coordinates.

21. (a) Average rate of change $= \frac{\Delta y}{\Delta x} = \frac{13 - 4}{2 - 1} = 9$
 (b) Average rate of change $= \frac{\Delta y}{\Delta x} = \frac{n - k}{m - j}$
 (c) The average rate of change is

$$\begin{aligned} \frac{\Delta y}{\Delta x} &= \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{(3(x+h)^2 + 1) - (3x^2 + 1)}{(x+h) - x} \\ &= \frac{(3(x^2 + 2xh + h^2) + 1) - (3x^2 + 1)}{h} \\ &= \frac{3x^2 + 6xh + 3h^2 + 1 - 3x^2 - 1}{h} \\ &= \frac{6xh + 3h^2}{h} \\ &= 6x + 3h. \end{aligned}$$

22. (a) The number of sunspots, s , is a function of the year, t , because knowing the year is enough to uniquely determine the number of sunspots. The graph passes the vertical line test.
 (b) When read from left to right, the graph increases from $t = 1945$ to approximately $t = 1947$, from approximately $t = 1954$ to $t = 1957$, and from approximately $t = 1964$ to $t = 1969$. Thus, s is an increasing function of t on the approximate intervals $1945 < t < 1947$, $1954 < t < 1957$, $1964 < t < 1969$. For each of these intervals, the average rate of change on any subinterval must be positive.
23. (a) Since Δt refers to the change in the numbers of years, we calculate

$$\Delta t = 1970 - 1960 = 10, \quad \Delta t = 1980 - 1970 = 10, \quad \text{and so on.}$$

Since the entries in the table are all 10 years apart, we see that $\Delta t = 10$ for all consecutive entries.

- (b) Since ΔG is the change in the amount of garbage produced per year, for the period 1960-1970 we have

$$\Delta G = 120 - 90 = 30.$$

Continuing in this way gives the Table 1.4:

Table 1.4

Time period	1960-70	1970-80	1980-90	1990-2000
ΔG	30	30	55	29

- (c) Not all of the ΔG values are the same. We know that all the values of Δt are the same. If we knew that all the values of ΔG were the same, we could say that $\Delta G/\Delta t$, the average rate of change in the amount of garbage produced each year, is constant. Since, on the contrary, ΔG is not constant, we conclude that $\Delta G/\Delta t$ is not constant. This tells us that the amount of garbage being produced each year is changing, but not at a constant rate.