Homework 9

Geometric Topology

Math 99r – Harvard University

Due Monday, 24 November 2003

1. Email me (`ctm@math.harvard.edu`) a brief outline of your final paper, including some references.

2. Find the absolute values of the linking numbers of the components of the links 4^2_1, 5^2_1, 6^2_1 and 7^2_5 (from Adams Appendix).

3. Let $T \cong K \times D^2 \subset S^3$ be a thickening of a knot K to a solid torus. A *longitude* for K is a knot $L \subset \partial T$ such that (i) L generates $\pi_1(T)$ and (ii) the linking number of L with K is zero.

   Draw longitudes for the unknot and the trefoil knot. In the case of the trefoil, find the element g represented by L in $\pi_1(S^3 - K) = \langle a, b : aba = bab \rangle$.

4. Let $L = A \subset B$ be a link with two components with linking number n. Show $[B]$ represents the class $\pm n \in H_1(S^3 - A, \mathbb{Z}) \cong \mathbb{Z}$.

5. What is the genus of the surface in Adams Figure 4.24?

6. Identify the surfaces in Adams Figure 4.46. (Are they both orientable? Give the genus, number of boundary components and Euler characteristic for each.)

7. Show the twist knots in Adams Figure 4.61 all have genus one. (How do you know these knots do not have genus zero?)

8. Show that if K is a knot given by a projection with s Seifert circles and c crossings, then Seifert's algorithm yields a surface of genus $g = (1+c-s)/2$.

9. Applying Seifert's algorithm, find surfaces bounded by the knots in Adams Figure 4.58, and identify their genus.

10. What is the first non-alternating knot in Adams Appendix? Apply Seifert's algorithm to find an upper bound on its genus.