Homework 8
Geometric Topology
Math 99r – Harvard University
Due Monday, 17 November 2003

1. Prove the $(1, n)$ torus knot is trivial for all $n > 0$.

2. Draw a picture of the $(7, 11)$ torus knot.

3. Using the Appendix, identify the prime factors of the composite knot shown in Adams Figure 1.15.

4. Express the 4 rational tangles in Adams Figure 2.23 in Conway notation, and determine which ones are equivalent.

5. Show that any algebraic knot or link which can be expressed in Conway notation with only positive integers is alternating.

6. Show that any rational tangle has an alternating projection (so the crossings alternate over and under along each strand).

7. Let L_n be the rational knot or link with Conway notation consisting of n 1's in a row $(111 \cdots 1)$.
   (a) What is the rational number associated to L_n? (You can use the word 'Fibonacci' in your answer.)
   (b) Using the Appendix, identify L_n for $n = 1, 2, 3, 4, 5$.
   (c) Find an efficient way to draw L_n in general, using a 3-strand braid, and use it to draw L_{10}.

8. Let T be a rational tangle, and let T_h and T_v be the results of rotating T by 180 degrees along a horizontal (E-W) and a vertical (N-S) axis. Show that T, T_h and T_v are all equivalent tangles.

9. Let T be a rational tangle, and let T' be obtained by rotating T clockwise by 90°. Show that the rational numbers encoding T and T' satisfy $q(T') = -1/q(T)$.