

Homework 4

Geometric Topology

Math 99r – Harvard University

Due Monday, 20 October 2003

1. Give a triangulation of \mathbb{RP}^2 and show your triangulation is not orientable.
2. Show explicitly that the Klein bottle K (obtained from $S^1 \times I$ by suitably gluing the ends) is homeomorphic to the connected sum of two copies of \mathbb{RP}^2 .
3. Find a group of isometries G of \mathbb{R}^2 such that \mathbb{R}^2/G is homeomorphic to a Klein bottle K . Then show that the subgroup H of translations in G has index two, and that \mathbb{R}^2/H is a torus double-covering K .
4. Prove that if Σ covers a surface of genus g with degree d , then Σ has genus $dg - d + 1$.
5. Let G be a path-connected topological group: this means G is a Hausdorff topological space, and multiplication and inverse are continuous maps from $G \times G \rightarrow G$ and $G \rightarrow G$.
Show that $\pi_1(G)$ is abelian.
6. Find a 2-complex with $\pi_1(K) \cong \mathbb{Z}^3$.
7. Show that a torus T can be given a cell decomposition consisting of six triangles, each embedded into T . Show it cannot be decomposed into 5 or fewer embedded triangles. (Hint: it is not necessary to enumerate triangulations.)
8. Give an example of a gluing pattern for a 3-dimensional polyhedron P , such that faces are identified in pairs, but $K = P/\sim$ is not a 3-manifold. (Hint: the only potential problem is at a vertex.)