Complex Analysis Homework 2
Math 213 — Harvard University
Due 3 October 2000

1. For $a, b \in \mathbb{C}$, express $a\overline{b}$ in terms of the dot-product and cross-product of the vectors represented by a and b.

2. Let $f(z)$ be one-to-one and analytic on a neighborhood of the unit circle $S^1 \subset \mathbb{C}$. Relate $\int_{S^1} \overline{f(z)} f'(z)\, dz$ to the area of the region enclosed by $f(S^1)$.

3. Draw the preimage of the upper halfplane under the sine function, $\sin^{-1}(\mathbb{H})$. Find all analytic homeomorphisms $h : \mathbb{C} \to \mathbb{C}$ such $\sin(h(z)) = \sin(z)$. Find all the solutions to $\sin(z) = i$.

4. Compute df/dz and $df/d\overline{z}$ for the map $f : \mathbb{C} \to \mathbb{C}$ given in polar coordinates by $f(r, \theta) = (r^\alpha, n\theta)$, $\alpha > 0$, $n \in \mathbb{Z}$. (Hint: express $f(z)$ in terms of $|z|$ and z^n.)

5. Suppose $f(z)$ is analytic. Compute $(d/dz)(d/d\overline{z})|f(z)|^2$.

6. Show that if f and g are analytic on a region U, and $|f(z)|^2 + |g(z)|^2 = 1$, then f and g are constant functions.