

## Homework 12

### Math 101 – Sets, Groups and Knots

The problems below refer to the knots tabulated in the Appendix of Adams' *The Knot Book*, p.279.

1. Determine which of the the following knots can be tricolored:  $5_1$ ,  $6_1$ ,  $7_4$ ,  $9_1$ .
2. Determine the factor knots that make up the composite knot in Adams' Figure 1.15. (Give their names from the Appendix.)
3. Find the absolute value of the linking number for the following two-component links:  $4_1^2$ ,  $6_3^2$ ,  $7_6^2$  and  $8_3^2$ .
4. Show that the links  $6_1^3$  and  $6_3^3$  are not equivalent. (Hint: use linking numbers.)
5. Compute a presentation for the knot group  $G(K)$  where  $K$  is  $5_1$ .
6. Prove that  $G(K)$  is nonabelian by constructing a surjective map to  $D_5$ . (Every generator should be sent to a flip.)
7. Show that for any knot group  $G$ , there is a surjective homomorphism  $\phi : G \rightarrow \mathbb{Z}$ . (Hint: guess what  $\phi$  should be on the generators of  $G$ , then show the relations of  $G$  are mapped to 0 by  $\phi$ ).
8. Show how to change the crossings of the knot projection  $9_{33}$  in Adams' tables, leaving the diagram otherwise unchanged, to obtain a projection  $K'$  of the unknot.