

Homework 11

Math 101 — Sets, Groups and Knots

The problems below refer to the knots tabulated in the Appendix of Adams' *The Knot Book*, (p.279).

1. What familiar group has the presentation $G = \langle a : a^{15} = a^{21} = e \rangle$?
2. Prove the group $G = \langle a, b : a^2 = b^2 = e \rangle$ is isomorphic to the infinite dihedral group $D_\infty = \langle r, f : f^2 = e, rf = fr^{-1} \rangle$. (Hint: first show there is a unique homomorphism $\phi : G \rightarrow D_\infty$ satisfying $\phi(a) = f$ and $\phi(b) = rf$. Then show ϕ is an isomorphism by constructing its inverse.)
3. Draw the figure eight knot 4_1 , and switch one crossing (change it from over to under or vice-versa) to obtain a new knot projection K . Show K is the unknot by giving a sequence of Reidemeister moves to undo all its crossings.
4. Show how to switch one crossing of 5_2 so the result K is still knotted. What knot do you get? Use Reidemeister moves to justify your answer.
5. Let N be your student ID number (or any other random number with 9 digits). Draw the knot 9_{18} , and label the crossings $1, 2, \dots, 9$ from top to bottom. Make a new knot projection K by switching the i th crossing of 9_{18} whenever the i th digit of N is odd. (For example, if $N = 617495277$, then you would switch crossings 2, 3, 5, 6, 8 and 9.)
Now simplify K as much as possible, and locate an equivalent knot L in the knot tables. Show K and L are the same knot by describing Reidemeister moves that transform K to L .
6. Prove that the $(n, 1)$ -torus knot K_n (see Figure 1) is equivalent to the trivial knot. (Hint: use induction on n .)

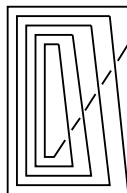


Figure 1. The $(5, 1)$ torus knot.