Throughout $X$ is a compact Riemann surface.

1. Let $\mathcal{Q}(X) \subset \mathcal{M}^1(X)$ denote the vector space of meromorphic 1-forms on $X$ with zero residue at every pole. Prove that:

$$H^1(X, \mathbb{C}) \cong \mathcal{Q}(X)/d\mathcal{M}(X).$$

2. Prove that a smooth $(1, 1)$-form $\alpha$ on $X$ can be expressed as $\alpha = d*df$ for some smooth function $f$ if and only if $\int_X \alpha = 0$.

3. Let $K$ be a canonical divisor on $X$. Express $\dim H^0(X, \mathcal{O}_{2K})$ in terms of the genus $g$ of $X$.

4. Suppose $\pi : X \rightarrow \hat{\mathbb{C}}$ presents $X$ as a 2-sheet covering of the sphere, branched over points $\{b_1, \ldots, b_{2n}\} \in \mathbb{C}$. Let $f : X \rightarrow X$ be the generator of $\text{Deck}(X/\hat{\mathbb{C}}) \cong \hat{\mathbb{C}}$.

(i) Show that every $\omega \in \Omega(X)$ satisfies $f^*\omega = -\omega$.

(ii) Given a pair of distinct points $p_1, p_2 \in X$, construct explicitly a meromorphic 1-form $\omega$ on $X$ with simple poles of residue $(-1)^i$ at $p_i$ and no other singularities.

5. Let $\pi : X \rightarrow \hat{\mathbb{C}}$ be a Riemann surface of genus two, presented as a 2-fold branched covering of the sphere. Prove that an effective divisor $D = P + Q$ is a canonical divisor iff $\pi(P) = \pi(Q)$.

6. Prove that for any divisor $D$ of degree $d$ on $X = \hat{\mathbb{C}}$, we have $h^0(D) = \max(0, 1 + d)$ and $h^1(D) = \max(0, -1 - d)$. 